

Rules and Problems for Interest and Loans in the Sanskrit Mathematical Texts

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The custom for taking interest is mentioned in the *Arthaśāstra* by Kautilya, a treatise on economic policy, military strategy and so on dated between 4th century BC and 2nd AD. This is the rule in book 3, chapter 11.

sapādapaṇā dharmyā māsavṛddhiḥ paṇaśatasya, pañcapanā vyāvahārikī, daśapaṇā kāntāragāṇām, viṃśatipaṇā sāmudrāṇām //

Translation by Datta and Singh in their *History of Hindu Mathematics* p.218: An interest of a paṇa and a quarter per month is just. Five paṇas per month per cent is commercial interest. Ten paṇas per month per cent prevails among forests. Twenty paṇas per month per cent prevails among sea traders.

Sanskrit texts available in print, their abbreviations and year completed.

AB: *Āryabhaṭīya* of Āryabhaṭa I. ca 499.

BSS: *Brāhmasphuṭasiddhānta* by Brahmagupta. 628.

BAB: Bhāskara I's commentary on the AB. 629.

PG: *Pāṭyaganīta* by Śrīdhara. ca.800.

GSS: *Gaṇitasārasaṃgraha* by Mahāvīra. ca.850.

MS: *Mahāsiddhānta* by Āryabhaṭa II. ca.950-1000.

SS: *Siddhāntaśekhara* by Śrīpati. fl.1039/1056.

GT: *Gaṇitatilaka* by Śrīpati.

L: *Līlāvati* by Bhāskara II. 1150.

GK: *Gaṇitakaumudī* by Nārāyaṇa. 1356.

The AB, BSS, MS and SS are books on astronomy that cover chapters on mathematics. AB 2, BSS 12, MS 14 and SS 13 include rules with no example. On AB Bhāskara gives examples with his own commentaries in his BAB. On BSS Pṛthūdaka (fl.864) gives examples, which are included in the edition by Dvivedi. Colebrooke's translation contains the English translation of examples. PG, GSS, GT, L and GK include examples. L and GK give answers to the examples in the auto-commentaries.

Rules and Examples

Notations: The interest is I for principal, P , in period, T , and the interest is i for the principal, p , in the period, t at the same rate.

The rule for five quantities first appeared in the BSS. The given quantities are set down for operation

$$\begin{array}{cc} T & t \\ P & p \end{array} \text{ or } \begin{array}{cc} P & p \\ T & t \end{array} . \text{ And } I \text{ and } i \text{ are}$$
$$\begin{array}{cc} I & i \\ I & i \end{array}$$

transposed with each other. When one of six terms is unknown it is found by $TPi = tpI$. Bhāskara I explains the rule as a combination of two rules of three quantities in his BAB as follows: When P, T, I, p, t are known and i is required, i' such that $P : I = p : i'$ is obtained. Then, $T : i' = t : i$. $i = \frac{i't}{T} = \frac{t}{T} \times \frac{pI}{P} = \frac{ptI}{PT}$.

1. Two rules in one stanza

Brāhmasphuṭasiddhānta 12.14

kālaguṇitaṃ pramāṇaṃ phalabhaktaṃ vyeḱaguṇahataṃ kālaḥ/
svaphalayutarūpabhaktaṃ mūlaphalaikyam bhavati mūlam//14//¹

Translation: The principal multiplied by the time and divided by the interest and multiplied by the multiplier less one is the time.

The sum of the principal and the interest, divided by unity added to its interest, is the principal.

Rule 1. for calculating in what time $P + i$ will be n times when $p = P$.

$$\frac{TP}{I}(n - 1) = t$$

P P

Derivation. T t This is simple interest.

I (n-1)P.

Example by Pṛthūdaka = GT 125 and 126

$$P = 200, T = 1, I = 6, n = 3.$$

$$P = 20, T = 2, I = 5, n = 1\frac{1}{2}$$

Rule 2. for calculating p when $p + i$ is given.

$$p = \frac{p+i}{1+i'}, i' = It/PT \text{ is called 'its interest (svaphala)'}.$$

$$\text{Derivation. } \frac{p}{i} = \frac{PT}{It}. p = \frac{PT}{PT+It}(p + i) = \frac{p+i}{1+\frac{It}{PT}}.$$

Example by Pṛthūdaka $P = 100, T = 1, I = 5, t = 10, p + i = 36$.

Answer $p = 24$.

2. Rule to separate p, i and the commission of the surety etc.

PG

kālapramāṇaghātaḥ parakālahatāḥ phalādayaś caite/
svayutihṛtāṃ miśraḱuṇā bhavanti mūlādayaḥ kramaśaḥ//48//²

Translation: The product of the time and the principal [is calculated]. The interest of and so on are multiplied by other times. These divided by their sum, being multiplied by the mixture, become the principals and so on.

Example in PG

¹āryā

²āryā

māsenā śatasya phalaṃ pañcaiko bhāvyake 'rdham atha vṛttau/
lekhakapādo varṣe pañcādhikanavaśatī miśram//54//³

Translation: In a month (T), for one hundred (P), the interest is five (I), [commission] of the surety ($bhāvyaka$) one (i_1), the fee ($vṛtti$) [of the calculator] a half (i_2), [the fee] of the scribe a quarter (i_3), and the mixed amount in one year ($t = 12$) is nine hundred and five (p).

$P = 100, T = 1, I = 5, t = 12, p + i_1 + i_2 + i_3 = 905$ surety $i_1 = 1$, calculator $i_2 = 1/2$, scribe (lekhaka) $i_3 = 1/4$.

$$p = \frac{100 \times 1}{100 \times 1 + (5 + 1 + \frac{1}{2} + \frac{1}{4}) \times 12} \times 905 = 500$$

$$i = \frac{5 \times 12}{100 \times 1 + (5 + 1 + \frac{1}{2} + \frac{1}{4}) \times 12} \times 905 = 300$$

$$i_1 = \frac{1 \times 12}{100 \times 1 + (5 + 1 + \frac{1}{2} + \frac{1}{4}) \times 12} \times 905 = 60$$

$$i_2 = \frac{\frac{1}{2} \times 12}{100 \times 1 + (5 + 1 + \frac{1}{2} + \frac{1}{4}) \times 12} \times 905 = 30$$

$$i_3 = \frac{\frac{1}{4} \times 12}{100 \times 1 + (5 + 1 + \frac{1}{2} + \frac{1}{4}) \times 12} \times 905 = 15$$

another example in GT 114 (p.83)

māsaikena śatasya kovida phalaṃ pañcaikako bhāvyake
vṛttau drammadalaṃ ca lekhakakṛte tadvat turīyāṃśakaḥ/
māsair dvādaśabhiḥ sakhe navaśatīmiśraṃ ca pañcōttaraṃ
mūlādyaṃ vada miśrakavyavahṛtau yady asti te kauśalam//114//⁴

Translation published in the *Gaṇitabhāratī* vol.4 nos 3-4 1982: The fruit of hundred in a month was, oh learned man, five (as interest), one futurity, half dramma for profession, and likewise one-fourth for the scribe. In twelve months, the mixed amount was nine hundred and five. Tell, oh friend! If you have proficiency in the treatment of mixed quantities, what were the principal etc?

3. Type of Problem: An amount of money p is loaned at the rate-interest, I , for a principal, P , in a period, T . A debtor pays back m for every t' . This question requires the time t for the debtor to pay back the whole amount of the debt.

Rule GSS 6.57.

mūlaṃ svakālaguṇitaṃ phalaguṇitaṃ tatpramāṇakālābhyāṃ
bhaktaṃ skandhasya phalaṃ mūlaṃ kālaṃ phalāt prāgvat//57//⁵

³āryā

⁴śārdūlavikrīḍita

⁵āryā

Translation by Raṅgācārya: The capital lent out is multiplied by its time (of installment) and is again multiplied by the rate-interest; this product, when divided by the rate-capital and the rate-time, becomes the interest in relation to the installment. The capital (in the installment) and the time (of discharge of the debt) are to be made out) as before from (this) interest. GK miśra

skandhakakālakālāntarahīnaskandhena bhājite vitte/
skandhakakālaviguṇite niyataṃ nirmuktakālaḥ syāt//10//⁶

Translation: When [some amount of] money (*vitta*) is divided by the installment (*skandha*) diminished by the interest (*kalāntara*) in the the time of installment (*skandhakakāla*), [and] multiplied by the the time of installment, [the result] is the time of discharge (*nirmuktakāla*) [from the debt].

$$\frac{p}{m - \frac{t'pI}{TP}} t' = t.$$

Derivation of this equation may be as follows:

$\frac{pI}{TP}t + p = \frac{m}{t'}t$; hence

$$p = \left(\frac{m}{t'} - \frac{pI}{TP}\right)t = \frac{1}{t'}\left(m - \frac{t'pI}{TP}\right)t.$$

p denotes the principal, m the installment, t' the time of discharge.

Any rule for the interest in the fixed period i , $\frac{t'pI}{TP}$ is not given here. This

is calculated by the method same as above: $\begin{matrix} T & t' \\ P & p \\ I & i \end{matrix}$.

Example GSS 6.58

māse hi pañcaiva ca saptatīnām māsadvaye 'ṣṭādaśakaṃ pradeyam/
skandhaṃ caturbhiḥ sahitā tv aśītiḥ mūlaṃ bhavet ko nu vimuktikālaḥ//58//⁷

Translation by Raṅgācārya: The rate of interest is 5 for 70 per mensem; the (amount of the) installment to be paid is 18 in (every) 2 months; the capital lent out is 84. What is the time of discharge?

$$T = 1, P = 70, I = 5, p = 84, m = 18, t' = 2$$

$$\frac{84}{18 - \frac{2 \cdot 84 \cdot 5}{1 \cdot 70}} \times 2 = 28$$

Example GK miśra

māsena pañcakaśatena śataṃ daśonaṃ
dattaṃ kuṣaṃkaṭa ihāpy adhamarṇakāya/

⁶āryā

⁷indravajrā

māśadvayaṃ prati sakhe daśapañcayuktāt
skandhaṃ drutaṃ kathaya me parimuktakālam//22//⁸

Translation: One hundred minus ten is loaned to a debtor at the rate of five percent per month. In times of difficulty the debtor [pays] an amount of fifteen every two months. Tell me quickly, friend, the time [in which] he is discharged.

$$T = 1, P = 100, I = 5, t' = 2, p = 90, m = 15.$$

$$i = \frac{2 \times 90 \times 5}{1 \times 100} = 9, \frac{90}{15-9} \times 2 = 30.$$

4. Type of problem: in order to be free from a debt which amounts $p + i$, one has to pay m every t' month in totally t months. This is the rule to compute p . GK miśra

nirmuktakālavṛddhyā rūpasya hi saikayā hatena bhajet/
skandhakakālena ca gatakālaskandhāhatir mūlam//11//⁹

Translation: One should divide the product of the elapsed time and the rate of installment by the period of installment multiplied by the interest on the time of discharge plus one. [The result is] the principal.

$$\frac{tm}{\left(\frac{tI}{TP} + 1\right)t'} = p.$$

Derivation of this equation may be as follows: $p + i = \frac{m}{t'}t$; $p + p\frac{tI}{TP} = \frac{m}{t'}t$; $p\left(1 + \frac{tI}{TP}\right) = \frac{m}{t'}t$, where $\frac{tI}{TP}$ is called the interest for one monetary unit in the months of being free (*nirmuktakālavṛddhyā rūpasya*).

Example GK miśra

pañcakaśatena vittaṃ māśadvitayena sadalena/
skandhaḥ pañcadaśātha triṃśanmāsā vinirmuktaḥ//23//
kālas tv iha vada mūlaṃ kiṃ vṛddhiḥ kā ca yadi vetsi/¹⁰

Translation: [Some amount of] money (*vitta*) [is loaned] at the rate of five percent [per month]. Two and a half months is [the time for each installment]. The installment is fifteen [and] the time of discharge is thirty months. Tell what the principal is and what the interest is, if you know.

$$T = 1, P = 100, I = 5, t' = 2\frac{1}{2}, t = 30, m = 15.$$

$$\frac{tI}{TP} = \frac{30 \times 5}{1 \times 100} = \frac{3}{2}, 1 + \frac{3}{2} = \frac{5}{2}, t' \times \frac{5}{2} = \frac{25}{2}$$

$$\frac{30 \times 15}{\frac{25}{2}} = 72.$$

⁸vasantīlakā

⁹āryā

¹⁰3/2 gītis

This example asks the interest i also. However, the commentary does not give answer. It is 108. The total amount is $(15/\frac{5}{2}) \cdot 30 = 180$; $180 - 72 = 108$.

Cf. GSS 6.64 65.

5. Rule to find the time in which a debt will be paid back by equal monthly installments. PG 49-50.

īpsitakālopanaye mūlaṃ mūlāt pṛthak pṛthag jahyāt/
śeṣasya māsikaphalaṃ viśodhayen māsikopanayāt//49//¹¹
śeṣeṇa māsikaphalaṃ māsagṇaṃ māsaśeṣayug vibhajet/
labdhaṃ gatamāsayutaṃ dhanapraveśe bhavet kālaḥ//50//¹²

Translation: In order to obtain the desired time one should subtract the principal from the principal one after the other. One should subtract the monthly installment on the residue from [the amount] obtained monthly. [One should divide] the result in a month by the remainder multiplied by [the number of] the months [elapsed] and increased by the residue of the month (read *mūla* for *māsa*, principal). The quotient increased by [the number of] the months elapsed is the time of discharge.

In the note on his English translation Shukla mentions the reading *māsaśeṣayug* at 50b should be *mūlaśeṣayug*. If it so, this reading is also same to Nārāyaṇa's. The underlined portions are similar to the reading in GK miśra 14-15.

skandhakakālopanayān mūlaṃ mūlāt pṛthak pṛthag tyaktvā/
avaśiṣṭasya ca māsikaphalaṃ tyajen māsikopanayāt//14//

¹³
śeṣeṇa māsikaphalaṃ māsagṇaṃ mūlaśeṣayug vibhajet/
labdhaṃ gatamāsayutaṃ dhanasya nirmuktakālaḥ syāt//15//¹⁴

Translation: In order to obtain the time of installments, one should subtract the principal from the [preceding] principal one after the other. And one should subtract the monthly interest on the residue from [the amount] obtained monthly. From the remainder one should divide the result in a month multiplied by [the number of] the months [elapsed] and increased by the residue of the principal. The quotient increased by [the number of] the months elapsed is the time of discharge.

¹¹āryā

¹²āryā

¹³āryā

¹⁴āryā

When K. S. Shukla refers to the rule and example in his introduction to the edition of PG he mentions as follows: ‘This rule and example, too, do not occur in any work written before the tenth century A.D. and seem to be due to Śrīdharaċārya. The occurrence of this rule and a similar example in the *Gaṇitakaumudī* of Nārāyaṇa proves Śrīdhara’s influence on Nārāyaṇa. pp. xvi–xvii. Example in PG

pañcakaśatena dattaṃ rūpaśataṃ yasya kasyacid dhaninā/
grhaṃ ādattaṃ tasmāt pratimāsaṃ dviguṇaviṃśatyā//55//¹⁵
ṛṇamuktas tatkiyatā kālena sa bhaved ṛṇī samācakṣva/
dhaninaḥ praviśati vidvan kiṃ śuddasthānalābhena//56//¹⁶

Translation by Shukla: A rich man lent to somebody a sum of 100 *rūpas* at 5 per cent (per month simple interest) and from him took a house bearing a rent of 40 (*rūpas*) per month. Say, learned man, after how much time is the debtor relieved of his debt, and what does the rich man get by the gain of bare accommodation.

calculation in the anonymous commentary: debt: 100. $P = 100, T = 1, I = 5, p + i = 40$.

$$t = 1, p_1 = \frac{100}{105} \times 40 = \frac{20}{21} \times 40 = \frac{800}{21}. i_1 = \frac{5}{105} \times 40 = \frac{1}{21} \times 40 = \frac{40}{21}.$$

$$100 - \frac{800}{21} = \frac{1300}{21}. \quad \begin{array}{c} 1 \\ 5 \end{array} \quad \begin{array}{c} 1 \\ 0 \end{array} \quad 100 \quad \frac{1300}{21} \quad . \quad 0 = \frac{65}{21} \cdot \frac{65}{21} + \frac{1300}{21} = \frac{1365}{21} = 65$$

$$t = 2. \quad p_2 = \frac{100}{110} \times 40 = \frac{10}{11} \times 40 = \frac{400}{11}. \quad i_2 = \frac{10}{110} \times 40 = \frac{1}{11} \times 40 = \frac{40}{11}.$$

$$\frac{1300}{21} - \frac{400}{11} = \frac{5900}{231}. \quad \begin{array}{c} 1 \\ 5 \end{array} \quad \begin{array}{c} 2 \\ 0 \end{array} \quad 100 \quad \frac{5900}{231} \quad . \quad 0 = \frac{590}{231} \cdot \frac{590}{231} + \frac{5900}{231} = \frac{590}{23}.$$

$$\begin{array}{c} 1 \\ 5 \end{array} \quad \begin{array}{c} 1 \\ 0 \end{array} \quad 100 \quad \frac{5900}{231} \quad . \quad 0 = \frac{295}{231} \cdot 40 - \frac{295}{231} = \frac{8945}{231} \cdot \frac{295}{231} \times 2 + \frac{5900}{231} = \frac{6490}{231} \cdot \frac{6490}{231} \div$$

$$\frac{8945}{231} = \frac{6490}{8945} = \frac{1298}{1789} \text{ month} = 21 \text{ days } \frac{1371}{1789} \text{ day}$$

Answer: 2 months and 21 $\frac{1371}{1789}$ days.

Example in GK miśra

daśakaśatena tu dattaṃ śataṃ ca pañcāśad uttaraṃ dhaninā/
pratimāsaṃ ṛṇī saphalaṃ pañcāśat skandhakaṃ prayacchati ca//25//
anṛṇī kālena sakhe prajāyate kena kathayāśu/¹⁷

¹⁵āryā

¹⁶āryā

¹⁷gīti+12,15 mātrā

Translation: One hundred increased by fifty was loaned at the rate of ten percent [per mentem] by a rich man. Every month the debtor pays fifty including interest [as] an installment. O friend, tell quickly in how much time the debtor will be free from the debt.

Calculation explained in the commentary: debt 150. $P = 100, T = 1, I = 10, p + i = 50$.

$$\begin{array}{r}
 100 \quad p_1 \\
 1 \quad 1 \quad ; p_1 + i_1 = 50. \rightarrow p_1 = \frac{500}{11}, i_1 = \frac{50}{11}. 150 - \frac{500}{11} = \frac{1150}{11}. \\
 10 \quad i_1 \\
 100 \quad p_2 \\
 1 \quad 2 \\
 10 \quad i_2 \quad ; p_2 + i_2 = 50. \rightarrow p_2 = \frac{125}{3}, i_2 = \frac{25}{3}. \frac{1150}{11} - \frac{125}{3} = \frac{2075}{33} \\
 e \\
 100 \quad p_3 \\
 1 \quad 3 \quad ; p_3 + i_3 = 50. \rightarrow p_3 = \frac{500}{13}, i_3 = \frac{150}{13}. \frac{2075}{33} - \frac{500}{13} = \frac{10475}{429} \text{ less} \\
 10 \quad i_3 \\
 \text{than } 50. \\
 1 \quad 1 \\
 100 \quad 10475/429 \quad ; i = 2095/858. 50 - \frac{2095}{858} = \frac{40805}{858}. \\
 10 \quad i \\
 \frac{2095}{858} \times 3 + \frac{10475}{429} = \frac{27235}{858}. \frac{27235}{858} \div \frac{40805}{858} = \frac{5447}{8161}.
 \end{array}$$