

Positional and tabular notations in Sanskrit mathematical texts (VIIth-Xth century)
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January 6, 2012

The decimal place value notation

A definition by Āryabhaṭa (499 CE), and a commentary by Bhāskara (628 CE) (Shukla 1976, Shukla-Sarma 1976, Keller 2006)

Ab.2.2. *Ekam ca daśa ca śataṃ ca sahasraṃ tvayutaniyute
tathā prayutam|*
Koṭyarbudaṃ ca vṛndaṃ sthānāt sthānaṃ daśaguṇaṃ syāt||
One and ten and a hundred
And one thousand, now ten thousand and a hundred
thousand, in the same way a million|
Ten million, a hundred million, and a thousand mil-
lion.
A place should be ten times the < previous > place||

BAB.2.2. One sets forth the places of numbers for the sake of easiness. (...)

< As for: > ‘**One and ten and a hundred and a thousand**’. One, ten, a hundred and a thousand have the first, second, third, and fourth place. (...) *Ayuta* has the fifth place. *Ayuta* is ten thousand. *Niyuta* has the sixth place. *Niyuta* is a *lakṣa* (a hundred thousand). ‘**In the same way**’ (*tathā*), that is, in exactly the same manner, *prayuta* has the seventh place. Ten *lakṣas* are a *prayuta* (**a million**). *Koṭi* has the eighth place. A hundred *lakṣas* are a *koṭi* (**ten million**). *Arbuda* has the ninth place. Ten *koṭis* are an *arbuda* (**a hundred million**). *Vṛnda* has the tenth place. A hundred *koṭis* are a *vṛnda* (**a thousand million**).

< As for: > ‘**A place should be ten times the < previous > place**’. Another place < should be > ten times the < previous > place; it amounts

to: the next place is ten times one's own ⟨ previously ⟩ singled out place.

⟨ Objection ⟩

For what purpose is this ⟨ fourth quarter of verse ⟩ stated? For certainly these places ⟨ given in the first three quarters ⟩ are ten times ⟨ in worth ⟩ in regard to the immediately adjoining ones. If the statement ⟨ in the fourth quarter is made ⟩ in order to understand places other than those ⟨ given in the first three quarters ⟩ (that is, the places coming afterwards) then the naming of places is useless.

(...)

⟨ Bhāskara ⟩

It should not be understood in this manner. (...) The names of numbers, beginning with one and ending with a thousand million, are fixed ⟨ by the first three quarters ⟩ . ⟨ With the fourth quarter ⟩ 'a place should be ten times the place ⟨ immediately before it ⟩ ', merely an assignment of places to a number beginning with one, is indicated, ⟨ and ⟩ not the names of numbers, because ⟨ the rule ⟩ is of no use ⟨ for naming numbers ⟩ .

Here this may be asked: What is the power (*śakti*) of the places, ⟨ that power with ⟩ which one unit becomes ten, a hundred, and a thousand? And truly if this power of places existed, purchasers would have shares in especially desired commodities. And according to ⟨ their ⟩ wish what is purchased would be abundant or scarce.¹ And if this was so, there would be the unexpected possibility for things to be different in worldly affairs (*lokavyavahāra*).

¹In other words, if the place decided the value, in the world as well as in the treatise, one could buy a small amount, and then increase it afterwards, by simply changing its place.

The *Pāṭīganīta* of Śrīdhara (ca. 8th century CE)

(Shukla 1959, Keller 2006b, Keller 2010)

PG.7-8 *ekaṃ daśaśataṃ asmāt sahasraṃ ayutaṃ tataḥ paraṃ lakṣam|*
arbudam abjaṃ kharvaṃ nikharvaṃ ca ||7||

tasmān mahāsarojaṃ śaṅkuṃ saritām patiṃ tatas tv antyam|
madhyaṃ parārddham āhur yathottaraṃ daśaguṇaṃ tajjñāḥ||8||

One, ten, a hundred, a thousand, ten thousand, and after a hundred thousand|

A million, ten millions, a hundred millions and a thousand millions ||7||

A billion, ten billions and a hundred billions and then a million billions|

Ten million billions and a hundred million billions are stated to be ten times the preceding by those who know them||8||

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