Operations and languages
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Seminar HPM – “Questioning operations. Writing calculations and operations: Language
pointers and environment”

**IM 54478**
OB tablet from Tell Harmal, published by Baqir (1951, Sumer 7, p. 30).
Single column landscape oriented tablet, containing one problem with detailed solution.
Calculation of the edge of a cube-shaped excavation, the volume being given.

**Transliteration**
**Obverse**
1. šum-ma ki-a-am i-ša-al-ka um-ma šu-u₂-ma
2. ma-la uš-ta-am-hi-ru u₂-ša-pi₂-il-ma
3. mu-ša-ar u₂ zu-u₂₄ mu-ša-ri
4. e-pe₂-ri a-su-uh ki-ia uš-tam-hi-ir
5. ki ma-si₂ u₂-ša-pi₂-il
6. at-ta i-na e-pe₂-ši-ka
7. [1.30 u₃] 12 lu-pu-ut-ma i-gi 12 pu-ṭu₂-ur-ma
8. [5 ta-mar 5 a-na 1].30 e-pe₂-ri-ka

**Reverse**
1. i-ši-ma 7.30 ta-mar 7.30
2. mi-nam ib₂-si₈ 30 ib₂-si₈ 30 a-na 1
3. i-ši-ma 30 ta-mar 30 a-na 1 ša-ni-im
4. i-ši-ma 30 ta-mar 30 a-na 12
5. i-ši-ma 6 ta-mar 30 mi-it-ha-ar-ta-ka
6. 6 šu-pu-ul-ka

**Translation**
**Obverse**
1. If one asks you this:
2. As much as I made (the sides of the base) confront themselves, I dug (the depth), then
3. a mušarum and half a mušarum (= 1 ½ sar),
4. my volume, I removed. How much (the sides) I made confront themselves?
5. How much (the depth) I dug?
6. You, in your doing,
7. [1.30 and] 12 inscribe and the reciprocal of detach:
8. [5 you see. 5 to 1].30 your volume

**Reverse**
1. raise. 7.30 you see. Of 7.30
2. what is the cubic root? 30 is the cubic root. 30 to 1
3. raise: 30 you see. 30 to the second 1
4. raise: 30 you see. 30 to 12
5. raise: 6 you see. 30 is your side,
6. 6 is you depth.
CBS 12648, problem 2

Tablet from unknown provenance (probably southern Mesopotamia, early OB period), published by Hilprecht (1906).

Type M(2,2) tablet containing several problems with detailed procedure.

Only the second problem is quite well preserved. It is similar to the previous one: find the dimensions of a rectangular prism, its volume and two relations between its edges being given.

Obverse, col. 1, lines 5-20

5. 2 še igi-12 gal₂ [sahar] 2 še 12th [the volume]
6. 2/3-bi uš-a-kam sag 2/3 of the length is the width
7. šu-ri-a sag-ga₂-kam The half of the width is its depth.
8. bur₃-bi
9. uš-bi sag-bi Its length, it width,
10. u₃ bur₃-bi [en-nam] and its depth [what ?]
11. uš [sag] The length, [the width],
12. u₃ bur₃-bi and its depth
13. ub-te-gu₇-ma make hold, then
14. igi-bi e-du₃-ma its reciprocal detach, then
15. sahar-še₃ ba-e-il₂-ma to its volume raise, then
16. ib₂-sa₂ the cubic root of
17. 15.37.30 15.37.30
18. el₁-de₃ extract.
19. ib₂-sa₂ 15.37.30 The cubic root of 15.37.30

Col. ii

1. [2.30] [is 2.30]
2. [broken] […]

VAT 8547, #1

Mathematical tablet from unknown provenance published by Sachs (1952, BMT III: 153). The tablet is single-column (type S) and contains a list of 4 problems which ask to calculate the cubic roots of 27, 1.4, 2.5, 3.36 (answers are 3, 4, 5, and 6). The procedure is detailed; they use a method of factorization. The first problem is the following:

Obverse

1. ṛba₁-si 27 [mi]-nu-[um] The cube root of 27 is what?
2. ṛat₁-ta i-na 27 7.30 u₂-su₄-su₆-uh-ma You in your <doing>, from 27, 7.30 tear out, then
3. 26.52.30 te-zi-ib 7.30 ša ta-as-su₄₃ (sic)-hu 26.52.30 you leave. 7.30 that you tore out below 26.52.30 place:
5. 26.52.30 7.30 ba-si 7.30 mi-nu-um 30 26.52.30. The reciprocal of 7.30 detach: 8
6. igi 7.30 pu-tur-ma 8 8 to 26.52.30 {7.30} raise: 3.36
7. 8 a-na 26.52.30 7.30 il₂ 3.36 The cube root of 3.36 is what? 6
8. ba-si 3.36 mi-nu-um 6 6 to 30 the cube root (of line 5) raise 3
9. 6 a-na 30 ba-si il₂ 3 The cube root of 27 is 3.
YBC 4708, #1-2
Tablet from unknown provenance (probably central Mesopotamia, end of the OB period), published by Neugebauer (MKT I p. 389 ss.)
The type is M(3/3). The tablet contains a list of 60 statements dealing with piles of bricks, with no procedure.
It ends with a colophon providing the number of sections and the serial number (1 šu-ši im-šu / [dub] 1-kam-ma = 60 sections / tablet number 1). This means that this tablet is the first of a series of several numbered tablets.
The two first sections read as follows:

Face, col. i

Transliteration

Transcription by Thureau-Dangin (TMB: 194)

Translation

# li.
1 1. [sig4]-[anše] 5 ninda uš-bi
2. [1 ½] ninda sag 1/2 ninda sukud-bi
3. sig4-bi en-nam
4. sig4-bi 3(iku) GAN2 24 sar

2 5. sig4 sig4-anše 3(iku) GAN2 24 sar
6. 1 1/2 ninda sag 1/2 ninda sukud-bi
7. uš-bi en nam 5 ninda uš

[A pile of bricks.]

[1 ½] ninda its width, ½ ninda its depth.
Its (volume in) bricks is what?

The (volume in) bricks of a pile of bricks is 3(iku) GAN2 24 sar,
1 ½ ninda its width and ½ ninda its depth.
Its length is what? The length is 5 ninda.
Procedure text: YBC 4663 #4
Procedure text from unknown provenance (probably southern Mesopotamia).
The tablet (type S) contains 8 problems dealing with the excavation of a trench by workers and the cost of this work. Each problem includes a detailed procedure.

Transliteration
20. 9 gin₂ ku₃ ki-la₂ 5 ninda uš 1 1/2 ninda sag 10 <gin₂> sahar eš₂-gar₃ 6 še a₂-bi♭
21. bur₃-bi- en-nam za-e in-da-zu-de₃
22. uš sag gu7-gu7-ta 7.30 i-na-di-ku igi eš₂-gar₃ pu-ṭu-ur
23. a-na 7.30 i-ši 45 i-na di-ku 45 a-na i-di i-ši
24. 1.30 i-na-di-ku igi 1.30 pu-ṭu-ur 40 i-na-di-ku
25. 40 a-na 9 ku₃ i-ši 6 bur₃-bi i-na-di-ku 1/2 ninda bur₃-bi <ki-a-am ne₂-pe-šu>

Translation
20. 9 gin₂ the (total expenses in) silver for a trench. 5 ninda the length, 1 1/2 ninda the width. 10 (gin₂) the assigned volume. 6 še (silver) the wage (per worker).
21. Its depth how much ? You, in your procedure :
22. The length and the width make hold. 7.30 will be given to you. The reciprocal of the assigned volume detach,
23. to 7.30 raise. 45 will be given to you. 45 to the wage raise.
24. 1.30 will be given to you. The reciprocal of 1.30 detach. 40 will be given to you.
25. 40 to 9, the (total expense in) silver, raise. 6, the depth, will be given to you. 1/2 ninda its depth.

Catalogue: YBC 4657, #4
YBC 4657 is a catalogue from unknown provenance (probably southern Mesopotamia). It contains 31 statements of problems dealing with the excavation of a trench by workers and the cost of this work. The tablet ends with a colophon providing the number of sections and the subject of the problems (label): 31 problems on excavations (31 im-šu ki-la₂).

Transliteration
8. ku₃ ki-[la₂ 9 gin₂] 5 ninda uš-bi 1 1/2 ninda sag 10 gin₂ eš₂-gar₃
9. 6 še a₂-bi lu₂-huḡ-ga₂ bur₃-bi en-nam [1/2 ninda] bur₃-bi

Translation
8. The (total expenses in) silver for a trench is 9 gin₂. 5 ninda the length, 1 1/2 ninda the width. 10 (gin₂) the assigned volume.
9. 6 še (silver) the wage (per worker). Its depth is what? [1/2 ninda] its depth.

<table>
<thead>
<tr>
<th>Groups in catalogue</th>
<th>Procedure texts</th>
<th>Mathematical content</th>
</tr>
</thead>
<tbody>
<tr>
<td>YBC 4657</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>#1-8</td>
<td>YBC 4663</td>
</tr>
<tr>
<td>II</td>
<td>#9-18</td>
<td>?</td>
</tr>
<tr>
<td>III</td>
<td>#19-28</td>
<td>YBC 4662</td>
</tr>
<tr>
<td>IV</td>
<td>#29-31</td>
<td>?</td>
</tr>
<tr>
<td>Colophon:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>31 sections on a trench</td>
<td>No colophon</td>
<td></td>
</tr>
</tbody>
</table>
AO 9071, #45


The type is M(3/3). The tablet contains a list of 95 statements dealing a rectangle. The tablets ends with a colophon providing the number of sections and the serial number (1(geš2) 35 im-šu / dub 7-kam-ma = 95 sections / tablet number 7). This means that this tablet is the seventh of a series of several numbered tablets. Other tablets of the same series are probably AO 9072, YBC 4695, YBC 4711.

Obverse, col. ii, 12th section (E₁)

35 15. uš a-ra₂ [3-e] tab
16. sag a-ra₂ 2-[e] tab
17. gar-gar-ma igi-13-gal₂-bi
18. uš dah-ma [40]

The length 3 times repeated, the width 2 times repeated, I accumulated, its 13th, to the length I added : 40.

[…]

Obverse, col. ii, last section (P)

37 25. uš 5 ninda bi₂-dah
26. igi-7-gal₂-bi u₃ 1(geš₂) ninda zi
27. igi-11-gal₂-bi a-ra₂ 6-e tab
28. 1.35 ba-zi
29. igi-13-gal₂-bi

The length to 5 ninda I added, its 7th from 60 ninda I subtracted its 11th 6 times repeated from 1.35 I subtracted its 13th,

[…]

Obverse, col. iii, 5th section (S)

42 5. a-na uš ugu sag diri
6. dah-ma 15

to what the length exceeds the width I added : 15.

[…]

Obverse, col. iii, 8th section (relation between P and S)

45 9. a-ra₂ 2-e tab ba-sa₂

2 times repeated: I made equal.

AO 9072, #29, l. 1-3

uš a-ra₂ 3 sag a-ra₂ 2-e tab

gar-gar igi-13-gal₂-bi

I accumulated (gar-gar), its 13th
daš₃ dah-ma 20
to the surface I added (dah): 20

\[(uš×3 + sag×2) \frac{1}{13} + uš×sag = 20\]

<table>
<thead>
<tr>
<th>uš</th>
<th>→</th>
<th>U</th>
<th>→</th>
<th>…</th>
</tr>
</thead>
<tbody>
<tr>
<td>sag</td>
<td>→</td>
<td>20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>aša</td>
<td>→</td>
<td>10</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Attested constructions in AO 9071 and AO 9072

**Arguments**

<table>
<thead>
<tr>
<th>Argument</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>uš</td>
<td>length</td>
</tr>
<tr>
<td>sag</td>
<td>width</td>
</tr>
<tr>
<td>a-ša₃</td>
<td>surface</td>
</tr>
<tr>
<td>ib₂₂ taka₄</td>
<td>remainder</td>
</tr>
</tbody>
</table>

**Relations**

<table>
<thead>
<tr>
<th>Relation</th>
<th>Description</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>A B ba-sa₂</td>
<td>A and B I made equal</td>
<td>A = B</td>
</tr>
<tr>
<td>A-ma B</td>
<td>A : B</td>
<td>&quot;</td>
</tr>
<tr>
<td>A-ma ugu B C diri</td>
<td>A exceeds B of C</td>
<td>A = B + C</td>
</tr>
<tr>
<td>A-ma B C ba-la₂</td>
<td>A is below B of C</td>
<td>A = B – C</td>
</tr>
</tbody>
</table>

**Opérations**

<table>
<thead>
<tr>
<th>Operation</th>
<th>Description</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>A B-še₃ bi₂₂-dah</td>
<td>A to B I added (heterogeneous addition)</td>
<td>A + B</td>
</tr>
<tr>
<td>A B dah</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td>A B gar-gar</td>
<td>A B I accumulated (homogeneous addition)</td>
<td>&quot;</td>
</tr>
<tr>
<td>A u₃ B</td>
<td>A and B</td>
<td>&quot;</td>
</tr>
<tr>
<td>A B</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td>A B ta ba-zí</td>
<td>A, from B I subtracted</td>
<td>B – A</td>
</tr>
<tr>
<td>A B ba-zí</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td>A B ba-zí</td>
<td>from A, B I subtracted</td>
<td>A – B</td>
</tr>
<tr>
<td>A B C ba-zí</td>
<td>from A and B, C I subtracted</td>
<td>A + B – C</td>
</tr>
<tr>
<td>a-na A ugu B diri</td>
<td>ce dont A excède B</td>
<td>A – B</td>
</tr>
<tr>
<td>A ugu B diri</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td>A a-ra₂ N-e tab</td>
<td>A, N times repeated</td>
<td>A×N</td>
</tr>
<tr>
<td>a-ra₂ N A</td>
<td>N times A</td>
<td>N×A</td>
</tr>
<tr>
<td>A igi-N-gal₂-bi</td>
<td>A, its Nth part</td>
<td>A \frac{1}{N}</td>
</tr>
<tr>
<td>igi-N-gal₂ A</td>
<td>the N part of A</td>
<td>\frac{1}{N} A</td>
</tr>
<tr>
<td>šu-ri-a A</td>
<td>the half of A</td>
<td>\frac{1}{2} A</td>
</tr>
<tr>
<td>A \frac{2}{3}-bi</td>
<td>A, its \frac{2}{3}</td>
<td>A\frac{2}{3}</td>
</tr>
<tr>
<td>\frac{2}{3} A</td>
<td>\frac{2}{3} of A</td>
<td>\frac{2}{3} A</td>
</tr>
</tbody>
</table>