

Operations and languages

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Seminar HPM – “Questioning operations. Writing calculations and operations: Language pointers and environment”

IM 54478

OB tablet from Tell Harmal, published by Baqir (1951, Sumer 7, p. 30).
Single column landscape oriented tablet, containing one problem with detailed solution.
Calculation of the edge of a cube-shaped excavation, the volume being given.

Transliteration

Obverse

1. *šum-ma ki-a-am i-ša-al-ka um-ma šu-u₂-ma*
2. *ma-la uš-ta-am-hi-ru u₂-ša-pi₂-il-ma*
3. *mu-ša-ar u₃ zu-u_z₄ mu-ša-ri*
4. *e-pe₂-ri a-su-uh ki-ia uš-tam-hi-ir*
5. *ki ma-ši₂ u₂-ša-pi₂-il*
6. *at-ta i-na e-pe₂-ši-ka*
7. *[1.30 u₃] 12 lu-pu-ut-ma i-gi 12 pu-ṭu₂-ur-ma*
8. *[5 ta-mar 5 a-na 1].30 e-pe₂-ri-ka*

Reverse

1. *i-ši-ma 7.30 ta-mar 7.30*
2. *mi-nam ib₂-si₈ 30 ib₂-si₈ 30 a-na 1*
3. *i-ši-ma 30 ta-mar 30 a-na 1 ša-ni-im*
4. *i-ši-ma 30 ta-mar 30 a-na 12*
5. *i-ši-ma 6 ta-mar 30 mi-it-ha-ar-ta-ka*
6. *6 šu-pu-ul-ka*

Translation

Obverse

1. *If one asks you this:*
2. *As much as I made (the sides of the base) confront themselves, I dug (the depth), then*
3. *a mušarum and half a mušarum (= 1 ½ sar),*
4. *my volume, I removed. How much (the sides) I made confront themselves?*
5. *How much (the depth) I dug?*
6. *You, in your doing,*
7. *[1.30 and] 12 inscribe and the reciprocal of detach:*
8. *[5 you see. 5 to 1].30 your volume*

Reverse

1. *raise. 7.30 you see. Of 7.30*
2. *what is the cubic root? 30 is the cubic root. 30 to 1*
3. *raise: 30 you see. 30 to the second 1*
4. *raise: 30 you see. 30 to 12*
5. *raise: 6 you see. 30 is your side,*
6. *6 is you depth.*

CBS 12648, problem 2

Tablet from unknown provenance (probably southern Mesopotamia, early OB period), published by Hilprecht (1906).

Type M(2,2) tablet containing several problems with detailed procedure.

Only the second problem is quite well preserved. It is similar to the previous one: find the dimensions of a rectangular prism, its volume and two relations between its edges being given.

Obverse, col. 1, lines 5-20

5.	2 še igi-12 gal ₂ [sahar]	2 še 12th [the volume]
6.	2/3-bi uš-a-kam sag	2/3 of the length is the width
7.	šu-ri-a sag-ga ₂ -kam	The half of the width is
8.	bur ₃ -bi	its depth.
9.	uš-bi sag-bi	Its length, its width,
10.	u ₃ bur ₃ -bi [en-nam]	and its depth [what ?]
11.	uš [sag]	The length, [the width],
12.	u ₃ bur ₃ -bi	and its depth
13.	ub-te-gu ₇ -ma	make hold, then
14.	igi-bi e-du ₈ -ma	its reciprocal detach, then
15.	sahar-še ₃ ba-e-il ₂ -ma	to its volume raise, then
16.	ib ₂ -sa ₂	the cubic root of
17.	15.37.30	15.37.30
18.	e ₁₁ -de ₃	extract.
19.	ib ₂ -sa ₂ 15.37.30	The cubic root of 15.37.30

Col. ii

1.	[2.30]	[is 2.30]
	[broken]	[...]

VAT 8547, #1

Mathematical tablet from unknown provenance published by Sachs (1952, BMT III: 153).

The tablet is single-column (type S) and contains a list of 4 problems which ask to calculate the cubic roots of 27, 1.4, 2.5, 3.36 (answers are 3, 4, 5, and 6). The procedure is detailed; they use a method of factorization. The first problem is the following:

Obverse

1.	ṛba ₇ -si 27 [mi]-nu-[um]	The cube root of 27 is what?
2.	ṛat ₇ -ta i-na 27 7.30 u ₂ -su ^(sic) -uh-ma	You in your <doing>, from 27, 7.30 tear out, then
3.	26.52.30 te-zi-ib 7.30 ša ta-as-su ₂ ^(sic) -hu	26.52.30 you leave. 7.30 that you tore out
4.	ša-pa-al 26.52.30 gar-ra-ma	below 26.52.30 place:
5.	26.52.30 7.30 ba-si 7.30 mi-nu-um 30	26.52.30. The cube root of 7.30 is what ? 30
6.	igi 7.30 pu-ṭur-ma 8	The reciprocal of 7.30 detach: 8
7.	8 a-na 26.52.30 7.30 il ₂ 3.36	8 to 26.52.30 {7.30} raise: 3.36
8.	ba-si 3.36 mi-nu-um 6	The cube root of 3.36 is what? 6
9.	6 a-na 30 ba-si il ₂ 3	6 to 30 the cube root (of line 5) raise 3
10.	27-e 3 ba-si	The cube root of 27 is 3.

YBC 4708, #1-2

Tablet from unknown provenance (probably central Mesopotamia, end of the OB period), published by Neugebauer (MKT I p. 389 ss.)

The type is M(3/3). The tablet contains a list of 60 statements dealing with piles of bricks, with no procedure.

It ends with a colophon providing the number of sections and the serial number (1 šu-ši im-šu / [dub] 1-kam-ma = 60 sections / tablet number 1). This means that this tablet is the first of a series of several numbered tablets.

The two first sections read as follows:

Face, col. i

Transliteration

#	li.
1	1. [sig ₄]-[anše] 5 ninda uš-bi
	2. [1 1/2] ninda sag 1/2 ninda sukud-bi
	3. sig ₄ -bi en-nam
	4. sig ₄ -bi 3(iku) GAN ₂ 24 sar
2	5. sig ₄ sig ₄ -anše 3(iku) GAN ₂ 24 sar
	6. 1 1/2 ninda sag 1/2 ninda sukud-bi
	7. uš-bi en nam 5 ninda uš

Transcription by Thureau-Dangin (TMB: 194)

YBC 4708 ¹⁾

1

[ama]rum ²⁾ 5 NINDA šiddašu
[1 mišil] NINDA pātum mišil NINDA mēlūšu
libittašu minām
libittašu 3 ikū 24 sar

2

libitti amarim 3 ikū 24 sar
1 mišil NINDA pātum mišil NINDA mēlūšu
šiddašu minām 5 NINDA šiddum

Translation

#	li.
1	1. [A pile of bricks.] 5 ninda its length,
	2. [1 1/2] ninda its width, 1/2 ninda its depth.
	3. Its (volume in) bricks is what?
	4. Its (volume in) bricks is 3(iku) GAN ₂ 24 sar.
2	5. The (volume in) bricks of a pile of bricks is 3(iku) GAN ₂ 24 sar,
	6. 1 1/2 ninda its width and 1/2 ninda its depth.
	7. Its length is what? The length is 5 ninda.

Procedure text: YBC 4663 #4

Procedure text from unknown provenance (probably southern Mesopotamia).

The tablet (type S) contains 8 problems dealing with the excavation of a trench by workers and the cost of this work. Each problem includes a detailed procedure.

Transliteration

20. 9 gin₂ ku₃ ki-la₂ 5 ninda uš 1 1/2 ninda sag 10 <gin₂> sahar eš₂-gar₃ 6 še a₂-bi¹
21. bur₃-bi- en-nam za-e in-da-zu-de₃
22. uš sag gu₇-gu₇-ta 7.30 i-na-di-ku igi eš₂-gar₃ pu-*tu-ur*
23. a-na 7.30 i-ši 45 i-na di-ku 45 a-na i-di i-ši
24. 1.30 i-na-di-ku igi 1.30 pu-*tu-ur* 40 i-na-di-ku
25. 40 a-na 9 ku₃ i-ši 6 bur₃-bi i-na-di-ku 1/2 ninda bur₃-bi <ki-a-am ne₂-pe-šu>

Translation

20. 9 gin₂ the (total expenses in) silver for a trench. 5 ninda the length, 1 1/2 ninda the width. 10 (gin₂) the assigned volume. 6 še (silver) the wage (per worker).
21. Its depth how much ? You, in your procedure :
22. The length and the width make hold. 7.30 *will be given to you. The reciprocal of the assigned volume detach,*
23. *to 7.30 raise. 45 will be given to you. 45 to the wage raise.*
24. 1.30 *will be given to you. The reciprocal of 1.30 detach. 40 will be given to you.*
25. 40 to 9, the (total expense in) silver, *raise.* 6, the depth, *will be given to you.* 1/2 ninda its depth.

Catalogue: YBC 4657, #4

YBC 4657 is a catalogue from unknown provenance (probably southern Mesopotamia). It contains 31 statements of problems dealing with the excavation of a trench by workers and the cost of this work. The tablet ends with a colophon providing the number of sections and the subject of the problems (label): 31 problems on excavations (31 im-šu ki-la₂).

Transliteration

8. ku₃ ki-[la₂ 9 gin₂] 5 ninda uš-bi 1 1/2 ninda sag 10 gin₂ eš₂-gar₃
9. 6 še a₂-bi lu₂-huḡ-ga₂ bur₃-bi en-nam [1/2 ninda] bur₃-bi

Translation

8. The (total expenses in) silver for a trench is 9 gin₂. 5 ninda the length, 1 1/2 ninda the width. 10 (gin₂) the assigned volume.
9. 6 še (silver) the wage (per worker). Its depth is what? [1/2 ninda] its depth.

Groups in catalogue YBC 4657		Procedure texts	Mathematical content
I	#1-8	YBC 4663	Volume of a prism (direct and reverse problems), simple proportionality, homogeneous quadratic equations.
II	#9-18	?	Volume of a prism (direct and reverse problems), homogeneous and non-homogeneous quadratic equations.
III	#19-28	YBC 4662	Simple and double proportionality
IV	#29-31	?	Varia
Colophon: 31 sections on a trench		No colophon	

AO 9071, #45

AO 9071 is from unknown provenance (probably central Mesopotamia, end of the OB period), published in Proust 2009, "Deux nouvelles tablettes mathématiques du Louvre : AO 9071 et AO 9072." *Zeitschrift für Assyriologie* 99:167–232.

The type is M(3/3). The tablet contains a list of 95 statements dealing a rectangle. The tablets ends with a colophon providing the number of sections and the serial number (1(geš₂) 35 im-šu / dub 7-kam-ma = 95 sections / tablet number 7). This means that this tablet is he seventh of a series of several numbered tablets. Other tablets of the same series are probably AO 9072, YBC 4695, YBC 4711.

Obverse, col. ii, 12th section (E₁)

35	15.	uš a-ra ₂ [3-e] tab	The length 3 times repeated,
	16.	sag a-ra ₂ 2-[e] tab	the width 2 times repeated,
	17.	gar-gar-ma igi-13-gal ₂ -bi	I accumulated, its 13 th ,
	18.	uš dah-ma [40]	to the length I added : 40.

[...]

Obverse, col. ii, last section (P)

37	25.	uš 5 ninda bi ₂ -dah	The length to 5 ninda I added,
	26.	igi-7-gal ₂ -bi u ₃ 1(geš ₂) ninda zi	its 7th from 60 ninda I subtracted
	27.	igi-11-gal ₂ -bi a-ra ₂ 6-e tab	its 11th 6 times repeated
	28.	1.35 ba-zi	from 1.35 I subtracted
	29	igi-13-gal ₂ -bi	its 13 th ,

[...]

Obverse, col. iii, 5th section (S)

42	5.	a-na uš ugu sag diri	to what the length exceeds the width
	6.	dah-ma 15	I added : 15.

[...]

Obverse, col. iii, 8th section (relation between P and S)

45	9.	a-ra ₂ 2-e tab ba-sa ₂	2 times repeated: I made equal.
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AO 9072, #29, l. 1-3

uš a-ra ₂ 3 sag a-ra ₂ 2-e tab	The length 3 times repeated and the width 2 times repeated
gar-gar igi-13-gal ₂ -bi	I accumulated (gar-gar), its 13 th
a-ša ₃ dah-ma 20	to the surface I added (dah): 20

$$(u\check{s} \times 3 + sag \times 2) \frac{1}{13} + u\check{s} \times sag = 20$$

		...		U		...
uš	→				30	
sag	→				20	
aša	→				10	

Attested constructions in AO 9071 and AO 9072

Arguments

uš	length
sag	width
a-ša ₃	surface
ib ₂ - taka ₄	remainder

Relations

A B ba-sa ₂	A and B I made equal	$A = B$
A-ma B	A : B	"
A-ma ugu B C diri	A exceeds B of C	$A = B + C$
A-ma B C ba-la ₂	A is below B of C	$A = B - C$

Opérations

A B-še ₃ bi ₂ -dah	A to B I added (heterogeneous addition)	$A + B$	
A B dah	"	"	
A B gar-gar	A B I accumulated (homogeneous addition)	"	
A u ₃ B	A and B	"	
A B	"	"	
A B-ta ba-zi	A, from B I subtracted	$B - A$	
A B ba-zi	"	"	
A B ba-zi	from A, B I subtracted	$A - B$	(rare)
A B C ba-zi	from A and B, C I subtracted	$A + B - C$	
a-na A ugu B diri	ce dont A excède B	$A - B$	
A ugu B diri	"	"	
A a-ra ₂ N-e tab	A, N times repeated	$A \times N$	
a-ra ₂ N A	N times A	$N \times A$	(rare)
A igi-N-gal ₂ -bi	A, its N th part	$A \frac{1}{N}$	
igi-N-gal ₂ A	the N part of A	$\frac{1}{N} A$	(rare)
šu-ri-a A	the half of A	$\frac{1}{2} A$	
A $\frac{2}{3}$ -bi	A, its $\frac{2}{3}$	$A \frac{2}{3}$	
$\frac{2}{3} A$	$\frac{2}{3}$ of A	$\frac{2}{3} A$	(rare)