

Mathematical Sciences in the Ancient World

New Theoretical Approaches to the Sources and Socio-Political Issues of the Present Day

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The SAW project is dedicated to mathematical sources that have come down to us from the ancient world, specifically, though not exclusively, to the sources produced in Mesopotamia, China, and the Indian sub-continent.

The ambition of SAW is to develop new theoretical approaches to the history of ancient mathematics, in order to highlight a variety of practices in the fields that today too often are presented as homogeneous wholes, that is, "Mesopotamian mathematics", "Chinese mathematics", and "Indian mathematics". To this end, SAW intends to concentrate on the mathematical sources related to two activities: astral sciences and state administrations in charge of financial matters. The goal is to create resources that would promote a new historical representation of mathematics. Focus will be placed on three areas deemed important to this pursuit:

1. The definition and use of measuring units —The practice of problems
2. Astronomy and calendar—The practice of tasks
3. History and historiography of mathematical cultures

The wider impact of the project is to define an alternative to the widespread vision of mathematics. The outcome of the research will be made available to school teachers and thus will contribute to bridge the gap between the young people and mathematical sciences.

Mesopotamia in the contexts of SAW, goals and aims

Mesopotamian sources offer us a unique picture into the history of Mathematics in general, and administration in particular. The large number of economic and administrative allow us to see mathematics, especially metrology, in action. We can document the use (or non-use) of mathematical knowledge within different political, social, economic, and administrative milieus throughout ancient Mesopotamia's over 3000 year history. This allows the project to identify distinct mathematical cultures not only from a place to another, but from one administrative environment to another. With this in mind we can approach the history of cuneiform mathematics with several questions:



How uniform was the Mesopotamian mathematical tradition? In studying this tradition, Neugebauer and Sachs as well as by Thureau-Dangin ignored the provenance of the mathematical texts that formed the basis for their research. The problem of the provenance of mathematical texts was addressed in the publication of MCT to a relatively minute chapter written by Goetze (MCT, ch. IV). This, coupled with the imposition of modern algebraic thought and technique on ancient sources, led to a unified picture of "Babylonian Mathematics" that has influenced modern Assyriology. However, Goetze's study did open the door to further refinement of textual milieu, such as Hayrup 2002.

How accurate is this picture of homogeneity? Was education conducted along the same lines from school to school and city to city? Do administrators display the same practices as seen and possibly learned in schools? In cases when connections can be brought to light between the two, are there schools which are closer to administrative practices? Evidence at this point seems to show a remarkable continuity of traditions and the travelling of scribes from one environment to another. However, our evidence may show the impact of local singular practices on education in some local school (e.g. Mari and Me-turan, cf. Cavigneaux 1999 and Cavigneaux & Colonna d'Istria 2009) or milieu (e. g. Old-Assyrian merchants, cf. Michel 2008).

An important feature of this project will be to shape methods that ground our interpretation of ancient sources. With this in mind we will examine the modern history of our ancient source material, especially that of the collections of cuneiform sources found in museums, universities, and private collections all over the world. This will help us to better understand how the classifications and make-up of the modern cuneiform archives have influenced our understanding of cuneiform mathematics in general.



Results so Far: New Translations of Old Texts, YBC 4698

Tablet YBC 4698 provides an interesting overview of how the economic problems were approached in Old Babylonian mathematics. Although well preserved, the tablet has been only partly published. In MKT I (153), Neugebauer confesses that he is not able to publish the text. Thureau-Dangin improved the reading of some words, and provided transcription and translation of section 16, the rest of the text remaining quite obscure (Thureau-Dangin 1937: 89-90). The text contains 17 statements of problems on interest rates, prices and profit, and ends with a colophon which indicates that the tablet is the 3rd of a series (sub-, kam-ma). Thus, YBC 4698 belongs to the small collection of 20 mathematical series tablets known to date. The origin of this tablet is unknown. However, the series tablets probably date from the end of the Old Babylonian period (late 18th century or early 17th), and may come from central Mesopotamia (perhaps Sippar or Kish). We present here a few examples from the text to be published in CDLI later this year.

Statement 3, Obv. col. i, 7-11: Prices and Rates

3 sila ₃ -ta ₃ -sag	Per 3	
sila ₃ of premium oil		
1[ban] 2 sila ₃ -ta ₃ -giš	(and) per 1 ban, 2 sila ₃ of common oil	
common oil		
1[giš] gin ₃ ku ₃ -babbar sum	1 gin ₃ of silver I gave.	

The details provided in this statement are the rates for premium and common oil, that is, 1 gin₃ silver for both 3 sila₃ of premium oil and 1 ban, 3 sila₃ of common oil. This is stated through the use of -ta, which is understood as "per 1 unit" without explicitly stating the unit which is either an in-kind unit or an in-silver unit. Since it is qualified by an in-kind unit, -ta must refer to 1 gin₃ silver. We also see that one gin₃ is given for the entire exchange in line nine and that the amounts of common and premium oil in the exchange were equal.

However Statement 3 lacks a question, though from the details provided one can postulate a question: what are the values in-silver of the common and premium oil in equal quantities ("made equal")?

We can find the answer by again, converting the quantities into Sexagesimal Place Value Notation and placing the items in a table similar to the tables for Combined Market Rate Exercises presented in Fibberg 2007 pp. 157ff. In the table, all of our calculations are in bold.

	Rate in-kind	Rate in-silver	Value in-silver	Value in-kind (all equal)
Premium Oil	3	20	48	2.24
Common Oil	12	5	12	2.24
Total		25	1	

To begin with we place the rates in-kind and in-silver on the chart. Note that the in-silver rate is the reciprocal of the in-kind rate.

When we add the rates in-silver we come up with 25, the reciprocal of which is 2.24. As the total value in-silver is 1, thus 2.24 is the average of the values in-kind (which "made equal"). We can thus fill in the 4th column. With this we can find the silver value in column 3: multiply column four by column two to get column three.

The values we find for column four add up to exactly one, that is, the value of the entire exchange in-silver presented in line 9 of the statement. We then convert these back to metrological values and find for 2 1/3 sila, 4 gin₃ of premium oil 2/3 gin, 24 še silver is given and for the same amount of common oil 1/6 gin, 6 še silver is given.

Statement 6, Obv. col. ii, 1-5: Profit

30 še gur	30 gur grain.	
1(aš) še gur-ta sa ₃ -ma	Per 1 gur I bought (1 gin ₃ I paid).	
4(barga) še-ta ba ₃ -ra	per 4 barga of grain returned (sold) (1 gin ₃ I received).	
ku ₃ diri en-nam	The profit is how much?	
7 1/2 gin ₃ ku ₃ -babbar din	7 1/2 gin ₃ of silver is the profit.	

Statement six is an exercise in calculating profit from the sale of 30 gur of grain. It is interesting that the answer is provided in line five: 7 1/2 gin₃.

Line one states that the problem is dealing with 30 gur of grain. The exchange rate for the purchase is provided in line two, one gur per unit, while the resale rate is given in line three, four barga per unit. Note that -ta is again used to qualify these rates as per 1 gin₃ of silver.

To solve this problem we again convert into Sexagesimal Place Value Notation and place this data into a similar tabular format as was present in all Old Babylonian sources. (see Statement 3; Fibberg 2007, pp. 157ff; Nemet-Nejat 1993, pp. 17-19)

Note again, the silver rate, column two, is the inverse of the rate in-kind, column one.

	Rate in-kind	Rate in-silver	Value in-silver	Value in-kind (all equal)
Resale	4	15	37.30	2.30
Purchase	5	12	30	2.30
Difference			7.30	

To solve we multiply the values in column four by those in column two which gives us again the values in column three. Column three then is the value of the grain for both the resale and the original purchase. The profit is found when we subtract the value of the original purchase from that of the resale. In the end we come up with 7.30 which when converted into its metrological value in weight, is 7 1/2 gin₃, the profit in-silver.

This interpretation assumes that ba₃ (to return) here has the sense "to sell", that is, the contrary of to buy (reverse operation). Note the construction of ba₃-ra, which is different from that of sa₃-ma. The translation reflects this difference.

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