

Conference  
*Cultures of Computation and Quantification*  
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ABSTRACTS

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*Working on and with division in early China*

When compared with the mathematical manuscripts from the earliest decades of the Chinese empire recently excavated, the oldest mathematical book in Chinese handed down through the written tradition, *The Nine Chapters* (1<sup>st</sup> century C.E.), presents interesting differences. The texts of its procedures regularly make use of theoretical terms that are not found in the manuscripts, or at least not with the same theoretical meaning. Some of these terms are read by the commentators as related to the reasons why the procedures are correct: “make communicate 通 *tong*,” “equalize *tong*,” *lü* —a concept of number as defined relatively to other numbers. Others point out that a certain structure has been identified in a set of operations. This is the case primarily for one of the terms by means of which division is prescribed *chu*, which is the basis for a whole terminological complex absent from the earliest manuscripts. I do not mean to say that theoretical terms are completely absent from the manuscripts. The *Book of mathematical manuscripts* excavated from a tomb sealed *ca* 186 B.C.E. contains some such terms. Moreover, where they occur is interesting. However, the extent and the nature of the phenomenon cannot be compared to what we have in *The Nine Chapters*.

Interestingly enough, a great deal of these terms occur in relation to division or related procedures, predominantly the rule of three. These clues strongly suggest that division has been an important topic of reflection in some Chinese milieus between the 3<sup>rd</sup> century B.C.E. and the first century C.E. The hypothesis is supported by many other facts. Division, for instance, with closely related procedures like the rule of three, is a major topic in the manuscripts, in which many procedures are devoted to its various types of execution. One has to bear in mind that the terms to which division is applied were usually complex measured quantities. Division is also the only operation for which operands are designated by technical

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terms. Further, elementary operations, such as changes of units, essential to the practice of division can be correlated to theoretical discussions developed in the commentaries on *The Nine Chapters* and to the emergence of mathematical concepts.

The talk will suggest that the execution of division required mainly three basic elementary operations. On one of them, the transformation of the operands into the smallest possible integers with respect to a single measuring unit, there seems to me a great continuity of practice between the manuscripts and *The Nine Chapters*, the main difference being the introduction of technical terms to designate procedures. On the other two elementary operations, through which the result is produced bit by bit, we can perceive a break between the time when the known manuscripts were written and the time of the compilation of *The Nine Chapters*. I argue that this break may have been correlated to a change in the number system with which integers were written down with counting rods. *The Nine Chapters* bears witness to the emergence of a system of practice with the instrument of computation, centered on division and the then opposed operation of multiplication, a system that was bound to last for centuries.

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### *Measuring Grains in Early China*

The recent discovery and publications of several mathematical manuscripts in Chinese has entirely renewed the evidence available to describe mathematical activity in early China. It has also shed light on the practice of mathematics in the milieus related to some sectors of the local bureaucracy. On the one hand, the manuscripts found seem to have belonged to officials active at the county *xian* 縣 level [Peng Hao 彭浩 2001]. On the other hand, they provide evidence showing that the mathematical writings in question were connected to their official activity. This fact holds true for the other documents found in the same sets as the mathematical writings: edicts and statutes as well as documents connected to the daily activities of officials. In this context, it is interesting that in contrast to the first text handed down through the written tradition, that is, *The Nine Chapters* (1<sup>st</sup> century CE), topics related to grains occupy pride of place in these manuscripts. One must, however, qualify this assertion. If grains are only one of the important topics dealt with in *The Nine Chapters*, their treatment in this book is in close continuity with what manuscripts evidence in this respect.

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The central role given to grains in the manuscripts is not surprising. This fact can be easily explained by the part played by grains in the management of the bureaucracy at the time. However, the manuscripts reveal new features, or shed light on features that had not been well enough understood in *The Nine Chapters*, regarding the management of grains in early China. One of these features will be at the center of this presentation, that is, the complexity of the use of measuring units to assess amounts of grain. By contrast to contexts in which to a given type of goods there corresponds a fixed type of measuring units, we argue that the measurement of grains in early China, that is, in Qin and Western Han China, required the use of measuring units for length, volume, weight, capacity and value. In relation to this fact, the manuscripts contain a much richer set of statements than was previously known to allow conversion between these various units. How and in which phases of the process of managing grain were the different measuring units used? This is one of the topics addressed in the presentation.

This situation, and the quantities that needed to be handled in this context, perhaps went hand in hand with mathematical tasks that were rather complex for officials. By contrast, the mathematical management of grains in terms of the measuring units and types of quantities used appears to be simpler in *The Nine Chapters*, in which mainly capacity units were used, and also sometimes length, volume and value were mentioned. The talk will attempt to account for what is either the reflection of a change in the system of grains, or the reflection of the fact that *The Nine Chapters* is a different type of mathematical writing.

#### Bibliography

Peng Hao 彭浩 2001. *Zhangjiashan hanjian «Suanshushu» zhushi 張家山漢簡《算數書》注釋 (Commentary on the Book of mathematical procedures, a writing on bamboo strips dating from the Han and discovered at Zhangjiashan)*, Beijing: Science Press (Kexue chubanshe), 2001.

**Serafina Cuomo** (University of London)

#### *Numeracy in ancient Greece and Rome*

The paper will be a short survey of the work I have done so far as part of a research project on numeracy in ancient Greece and Rome. I will touch on issues of notation, calculating devices and procedures, measurement, and also talk briefly about numeracy in the wider contexts of administration and political life.

**Wolfgang Heimpel** (University of California, Berkeley)

***Ancient Mesopotamian brick and reed bundle metrology in theory and praxis***

One group among the administrative documents of the time of the Third Dynasty of Ur that ruled southern Mesopotamia at the end of the 21<sup>st</sup> century BCE, the so-called Garshana tablets, record the activities of the household of a high official and brother-in law of the reigning king. The records include the expenditures for ambitious construction projects over three years. An enclosure wall was built or rebuilt, and so was a flourmill, craftsmen's shops, other facilities, and residences. At the end of each day, the number of workers and the work done by them was written down. One particular work assignment was the transport of bricks from brickyard to construction site. Recorded were the number of brick carriers, of bricks carried, and the distance over which they were carried.

Old Babylonian texts written some 300 years later include lists of numerical constants that helped solving all kind of computational problems. A group of these constants was used by the ancient builder and administrator who wanted to know how many bricks of what kind would be needed and how many day wages the manufacture and transport of materials and the construction itself would require. I will demonstrate that the constants of the Old Babylonian texts were already used in the earlier Garshana texts.

The brick constants were based on numerically defined bricks. Comparisons with actual bricks have been done on the basis of the collection of baked and stamped bricks in the British Museum by E. Robson. They include good matches for the Old Babylonian period, less so for the time of the Garshana texts. M. Sauvage published the measurements of close some 280 bricks from that time. I will assess the difference between the numerically defined brick types in Garshana and actual bricks of the time on that basis.

The constants used in brick metrology describe bricks of different sizes as divergent from a standard brick. I will propose that reed bundle metrology provides another example of describing size differences with respect to a standard.

**Alexander R Jones** (ISAW)

***Mathematics and Metrology in a Fourth-Century Greek Papyrus Codex***

Twelve leaves from a fourth-century papyrus codex containing mathematical problems, metrological texts, and model contracts written in Greek survive, divided between a private collection and the Cotsen Children's Library (Princeton). An edition by Roger Bagnall, Alexander Jones, and a team of scholars is nearing completion. The problems include much mensurational geometry, but also numerical problems, some of which remain obscure. The typical format of a problem is, as in other Greek mathematical papyri, a brief statement of the presumed situation and the given quantities followed by a step-by-step algorithmic calculation of the desired quantity. In several cases the author of the solutions appears to have misunderstood the problem or to have applied an erroneous algorithm, and the diagrams accompanying the problems sometimes imply a misinterpretation of the geometrical

configuration to be assumed. In addition to these features and their implications for the didactic process and mode of composition of the manuscript, we will also look at the role of metrology in the problems and its relation to the metrological texts in the same codex as well as in other papyri.

**Agathe Keller & Catherine Singh** (SPHERE & SAW Project)

***Multiplying Integers: on the diverse practices of medieval Sanskrit authors***

Sanskrit medieval authors alluded to the fact that there could be several ways to execute multiplications with integers. In this paper/article we will examine the way in which medieval commentators of Brahmagupta (628 CE), Mahāvira (ca. 850) and Śridhara (ca. 950) diversely understood the ways a multiplication could be carried out. This study, providing previously un-edited and un-published material, will describe the contrasting practices of multiplication, highlighting the way different commentators shape numbers to execute this operation, and how such a procedure was laid out on a working surface. Such a study will also revisit the current historiography of elementary operations in Sanskrit sources.

**Takanori Kusuba** (Osaka University of Economics)

***Loans and Interest in the Līlāvātī of Bhāskara and in the Gaṇitakaumudī of Nārāyaṇa***

The *Gaṇitakaumudī* (1356) is most complete Sanskrit mathematical text after the *Līlāvātī* (ca.1150) in the field of *pāṭīganīta*. As for the topic 'interest' Bhāskara gives 3 rules and 3 examples. Nārāyaṇa, on the other hand, teaches more rules and examples together with those of loans. I compare the contents of the rules and the examples in the procedure called mixture (*miśra*) in the two major texts and investigate Nārāyaṇa's sources.

**Baptiste Mèlès** (Archives Henri Poincaré — Université de Lorraine)

***How to Program an Abacus?***

Computing tools do not only differ from a material point of view. They obey to various computing styles, which can be described with precision and generality with the help of some concepts of computer science.

Studying Chinese mathematical texts, we intend to show how the theory of programming languages can be used to describe stylistic differences between the Chinese counting rods and the Chinese abacus (suanpan). The notion of data structure will be shown to describe the tool itself, while the notion of programming paradigm describes the role of the hand.

**Cécile Michel** (ArScAn-HAROC (CNRS, Université Paris 1 Panthéon – Sorbonne & Université Paris Ouest Nanterre La Défense), SAW project)

***Weighing units and weights in the context of trade from Upper Mesopotamia to Anatolia (19<sup>th</sup>-18<sup>th</sup> centuries B.C.E.)***

The private archives of the Assyrian merchants found in central Anatolia highlight the use of the weight system, well adapted to quantify the metals they traded from East to West and back. An atypical progressive metrological list of weights found in a merchant's house at Kültepe/Kaneš shows the importance of fractions in the Old Assyrian numeracy: they better used the fraction of the superior unit than the smaller unit. This had certainly an impact on their computing practices. Account documents reveal two different cultures for the use of weighing units according to the quantified commodities and the geography. In Upper Mesopotamia, tin and silver were measured with the use of talents (gú), minas (mana) and shekels (gín), while in Anatolia, locally traded copper could be counted with or without the use of the talent (gú): large quantities were occasionally expressed in hundreds of minas. Also, merchants had to deal with different standards: the Anatolian mina weighted ten per cent less than the Assyrian mina. Texts mention controls of weights and reveal methods to correct weight deviations. A hundred and fifty stone weights found at Kültepe may be compared with the data provided by the metrological list, taking into account that there were at least two main standards for the mina. These weights, which show significant discrepancies, witness the existence of many individual weight standards, and confirm the recurrent necessity to correct weight deviations.

**Robert Middeke-Conlin** (SPHERE (CNRS & University Paris Diderot), SAW project)

***The Nazbalum in Old Babylonian Mesopotamia: A Concrete Number or Administrative Tool***

The nazbalum, a number related to brick deliveries and earth transportation, appears in coefficient lists of the Old Babylonian period while its use is apparent in mathematical problem texts of this period, though seldom explicitly stated in these problems. Prior explanations of the nazbalum have defined it as a concrete number of bricks transported over a month long period. However, a study of this coefficient's use in mathematical texts shows that the nazbalum is not a concrete number of bricks but is instead a coefficient used to define daily labor and appears on first inspection to be a fixed product of distance multiplied by the number of bricks which is never converted into a system of measure or quantification, nor defined by magnitude. It is an abstraction. This paper will examine how the nazbalum was used in calculations, that is, what is the nature of the mathematical problems in which the multiplication was acted out, as well as the potential context in which the nazbalum was used.

**Xiaoli Ouyang** (Fudan University, Shanghai, China) &  
**Christine Proust** (SPHERE (CNRS & University Paris Diderot) & SAW project)  
*Marginal Numbers and the Sexagesimal Place Value Notation in Mesopotamia during the Ur III Period (c. 2112-2004 BCE)*

Mesopotamia is well-known for the invention of the sexagesimal place value notation (hereafter SPVN), which appeared in full blossom in mathematical texts from the Old Babylonian period (c. 2000-1600 BCE). In a project sponsored by SAW, however, we have analyzed possible new evidence of the SPVN used in a different context—administrative records—from the earlier Ur III period. Our evidence comes from the so-called marginal numbers, which appear separated from the main text, either on the edges of a cuneiform tablet or in a distinct box on the obverse or reverse of a tablet. Such numbers tend to be isolated numbers, without indication of the metrological unit or name of the good. While some marginal numbers appear written in the regular numerical (the so-called S system) or metrological notation and represent subtotals in the main text, others seem to be denoted in other systems, including the SPVN, and fulfill a broader range of functions.

Our research has opened up a series of new questions about the diversity of the numerical notations detected in the marginal numbers, such as positional versus non-positional notation and sexagesimal versus non-sexagesimal. To begin with, did the Ur III positional notations exhibit the same features as the Old Babylonian SPVN? If not, how could we explain their difference? The next set of questions concern the relationship between the possible SPVN numbers and the numbers in the main text. Did the marginal numbers result from operations of addition or subtraction involving the numbers in the main text? Or were they meant for computations of the next step in the accounting process? Is there a connection between the notation of the marginal numbers and the operations carried out with these numbers? Last but not least, we may ask what kind of clues we could extract from such numbers regarding the tool and method of computation back to the Ur III period. Seeking an answer to these questions will shed new light upon the early development of the SPVN in Mesopotamia of the third millennium BCE.

**Alessandra Petrocchi** (SPHERE (CNRS & University Paris Diderot), SAW project)  
*Coins, seeds, and units of weight in the Arthaśāstra and in the commentary Kriyākramakarī*

Using seeds for weighing precious metals and stones was a common practice in ancient India. What then was the relation between coins, seeds, and units of weight?

In this regard, the *Arthaśāstra* (1st BCE - 1st CE), the first Sanskrit text on politics and governance that has come down to us, presents interesting details which are not found in later legal or mathematical texts. In which ways does the weight system described in this text differ from the one given in later literature? How are its heterogeneous features related to each

other? Is it possible in this context to identify common patterns? Also, in the *Arthaśāstra* the coinage system is treated in a separate section. How in this text were coins and units of weight related?

My paper will try to answer to these questions through a comparative study which takes into account, together with the *Arthaśāstra*, legal texts such as the *Manusmṛti* (2nd -3rd CE) and the *Yājñavalkyasmṛti* (3rd-5th CE) on the one hand, and the translation and analysis of these topics as found in the commentary on the *Arthaśāstra*, namely the *Cāṅkyaṭīkā* (before 10th CE?). Moreover, I will contrast such usages with those of a mathematical text in its treatment of weights and coins.

For this purpose, I will present passages selected from the *Līlāvātī* (12th CE) and one of its commentaries, the *Kriyākramakarī* (15th-16th CE).

**Christine Proust** (SPHERE (CNRS & University Paris Diderot) and SAW project)

***Shaping numbers and units of measure: some evidences in Early-Dynastic tables of surfaces from Southern Mesopotamia (mid-third millennium BCE)***

This contribution is an attempt to understand the methods of computation of surfaces developed in Mesopotamia before the use of sexagesimal place value notation, which appeared in cuneiform sources by the end of third millennium BCE (Ur III period). The documentation examined here is a set of tables of surfaces, considered by historians as the earliest known mathematical texts.

Five tables of correspondences between sides and surfaces of squares (or rectangles), dating from Early Dynastic periods (c.a. 2600-2350 BCE) are known to date. These documents provide information on the work on length and surface measures carried out by ancient scribes in order to solve mathematical problems linked to calculation of surfaces. In this talk, I propose to examine closely the organization of the tables and the notations of measures to highlight the way in which various metrological systems for surface were shaped around the mid-third millennium BCE in order to calculate the surface of small squares.

I shall show that:

The notion of surface, when applied to small squares, is a theoretical extension of a common notion of surface applied by surveyors to fields and lands.

The calculation of a small surface does not use the same methods as the calculation of a large surface.

The key issue for calculating small surfaces is the fact that there is only one bridge between length measures and surface measures.

Several different conceptions of multiplication emerge from the examination of early dynastic tables of surfaces.

**Stephanie Rost** (State University of New York at Stony Brook, USA)

***Computation and quantification in the administration of irrigation systems in 3rd millennium B.C. Mesopotamia***

Mesopotamian administrative documents provide an extraordinary insight into the mathematics used in the recording of economic data from various sectors of early state economies. This paper will discuss the computation and quantification procedures as related to the administration of large scale irrigation system in Mesopotamia at the 3rd Millennium B.C.

The largest and the most detailed corpus of such administrative texts derived from the highly bureaucratic Ur III state (2112-2004 B.C.) that controlled an area roughly corresponding to modern southern Iraq at the end of the 3rd millennium B.C.. Control was exercised by the division of the empire into administrative districts (provinces) ruled by separate military and civil authorities appointed by the king.

It has been proposed that Ur III bookkeeping served to keep track of the fulfillment of a province's tax obligations and/or was the result of a planned economy. Thus, Ur III documents seem to focus mainly on expenditures of labor and goods. Therefore, the information provided on irrigation management consist of a) amount of labor days expended for a specific irrigation task (i.e. cleaning canal, irrigating a field) and b) specifies earth volumes and the amounts and types of materials used in the construction and/or maintenance of these irrigation systems. Four work assignment texts (ASJ 19, Nr. 127 (BM 93831), CT 3, 35ff. (BM 21335), JCS 63, 35ff (HSM 6485), MVN IX 156) exemplify the computation practices underlying the administration of the maintenance of the so called i7-NINAKi-še3-DU canal; a primary canal that connected all four major cities of the Ur III province Lagash. These texts provide insight into one of the earliest examples of geographic survey practices by which means the natural environment was translated into quantifiable units, necessary to compute workload, labor demand and the division of the work amongst different social groups and state institutions.

Thus, these texts do not only provide an insight into mathematics the developed in relation to administration of the manipulation of the natural environment at the end of the 3rd millennium B.C, but also provides information on the historical geography as well as the socio-political organization of the Ur III State.

**Walther Sallaberger** (Munich University)

***Capacity measures in Early Mesopotamia***

Grain, the basic foodstuff of Mesopotamia, was measured by capacity, as were many other goods. Various systems of capacity measures were at use in the late Early Dynastic period (24th century BCE), until a standardized Babylonian system came to rise under the dynasty of Akkade (23rd century BCE). The contribution presents shortly the historical development and concentrates on the following issues:

- 1) Within the system both the "liter" (sila) and the bariga of 60 liters can be regarded as defining the standard.
- 2) How should we imagine measuring devices?
- 3) What is the relationship to the so-called "capacity measures" of liquids?

How is the system of capacity measures linked to other systems of measure, both in absolute size and in value?

**S. R. Sarma** (Duesseldorf)

***Grades of Gold and the related Computational Methods in Indian Texts***

The monetary value of a gold coin was based, in pre-modern times, on its intrinsic value, i.e. the amount of pure gold contained in it. Assayers and jewellers tested the degree of purity or fineness of gold coins and gold jewels and determined their price accordingly. Sanskrit mathematical texts teach how to calculate the proportion of gold and base metal in an alloy or how to exchange certain quantity of gold of fineness  $x$  against another piece of gold of fineness  $y$ , and similar problems. My paper will describe the different scales of fineness of gold that were prevalent in India and discuss the computational methods related to these.

**Zhu Yiwen** (SPHERE (CNRS & University Paris Diderot), SAW project)

***Another Culture of Computation from 7<sup>th</sup> century China***

In the early Tang period, many scholars commented on Confucian canons. Their commentaries bear witness to another culture of computation that was different from the one evidenced in the books that were selected to become an anthology of mathematical canons precisely at the same time period. This paper aims at characterizing this culture of computation by analyzing the sources found in Confucian canons and comparing them to the sources provided by mathematical canons.

The most conspicuous characteristic feature of that other culture of computation was the construction of specific procedures of computation. Unlike the procedures in mathematical books, those procedures of computation did not use any of the mathematical terminology that indicates the practical operation of counting rods. Moreover, these procedures of computation were full of reasoning, by means of which 7<sup>th</sup> century scholars not only built a bridge between the procedures and multiplication table and other tables, but also built a bridge between the procedures and their corresponding geometrical meanings. The two bridges appeared with different procedures, depending on which specific roles the measured quantities played in different cases. Moreover, slight differences can be perceived among procedures found in different Confucian canons.

Looking at these sources from the viewpoint of the social factors helps us to understand the reason that caused the fact that there existed a gap between these two cultures of computation.

From this point of view, this gap remained from this early period till the 19<sup>th</sup> century. Such a remark provides a strong incentive to rethink the history of mathematics in China.

**ZOU Dahai 鄒大海** & **CHEN Wei 陳巍** (Institute for the History of Natural Sciences, Chinese Academy of Sciences)

***The Characteristics of Calculation Methods in the Wu Cao Suan Jing and its Social Background***

From the 3<sup>rd</sup> to the 7<sup>th</sup> century CE, mathematics in China developed rapidly. However, this development is not reflected in the *Wu Cao Suan Jing* (五曹算经 *Computational Canon of the Five Administrative Sections*, hereafter *Wu Cao*). The most advanced methods contained in other writings appear to have been excluded from this book, compiled in the 6<sup>th</sup> century. The *Wu Cao* deals with a greater number of shapes and introduces more names of fields, with corresponding formulae for their areas, but it adopts less advanced methods of lower precision. It deliberately avoids fractions, and prefers specific methods rather than abstract ones. When the problems need to execute divisions which would have remainders, to express the results, the book uses smaller decimal units rather than fractions, which, however, occur in most other mathematical documents. All these features of *Wu Cao* can be explained by social factors, such as the need of training lower-grade officials of local governments who had to deal with complex managerial questions involving an important amount of calculations, in particular in relation to the need of enforcing the Equal-Field System which was enacted in 485 CE. Mathematical knowledge can thus be influenced by social factors, as much as the authors' ability in mathematics and their individual interests. The choice of methods of the *Wu Cao* and the way in which *Wu Cao* expresses mathematical knowledge, definitely betrays the influence of social factors.

## MEASURING UNITS

### Measuring Units in India

#### Silver and gold weights

#### IN MANUSMṚTI 8.132-137 AND YĀJÑĀVALKYA 13.363-365

The system of weights and coins of ancient and medieval India presents a great variety. One often finds that the same term in one Sanskrit text has a different value in another text, or that different terms correspond to the same value or different terms for different values. One should certainly consider local usages, differences in the ratio between precious metals, and that seeds/grains (associated to units of weight) have not always a uniform size. Thus, it is not possible to identify only ‘one standard system’.

The silver and gold weight system described in the Sanskrit text *Arthasāstra* (1 BCE-1 CE?) presents interesting details which are not found in later literature. The *Manusmṛti* (2<sup>nd</sup> -3<sup>rd</sup> CE) and the *Yājñavalkya* (3<sup>rd</sup> -5<sup>th</sup> CE) two later legal texts belonging to the *Dharmaśāstra* tradition, share an identical silver and gold weight denominations and values.

The close relation between the coinage and the weight-system of a country is well known. The very names of the coins in ancient India indicate this connection, since many of them derive from the name of their corresponding weights. For instance, in the *Arthasāstra* one finds the silver and gold weight denomination *māṣaka*, if it is silver *māṣaka* is equal to 1/16<sup>th</sup> of a silver *dharaṇa*, and if it is a *māṣaka* of gold is equal to 1/16<sup>th</sup> of a gold *suvarṇa*. Moreover, it is mentioned in this text as a copper coin. This has led some scholars to confusing conclusions.

*Manusmṛti* and *Yājñavalkya* weights:

the *guṇjā* is a seed of the *Abrus Precatorious*, it is very common in India and very often also used to weight spices and medicinal herbs. Here it is the standard unit, which forms the basis for weight denominations.

#### 1. SILVER

dharaṇa/ purāṇa ← 16 ← māṣaka ← 2 guṇjās

#### 2. GOLD

dharaṇa ← 10 ← pala ← 4 ← suvarṇa/karṣa 16 ← māṣa ← 5 guṇjās

In the *Arthaśāstra* the *gaurasarṣapa* ‘mustard seed’ is the basic unit for silver weights and in the mathematical text *Līlāvati* 12<sup>th</sup> CE (with its commentary *Kriyākramakarī* 15<sup>th</sup>-16<sup>th</sup> CE) the *valla* ‘barley corn’ has the same purpose.

## ARTHAŚĀSTRA

### 3 .SILVER

**dharāṇa** ←16 ←māṣaka ← mustard seeds

### 4.GOLD

**pala** ←4 ←suvarṇa /karṣa ←16 ←māṣaka ←10 māṣas  
5 guñjās

## LĪLĀVATĪ/ KRIYĀKRAMAKARĪ

### 5.SILVER

**gadyāṇa** ←2 ←dharāṇa ←8 vallas  
dhaṭaka ←14

### 6. GOLD

**tulā 100** ←pala ←4 ←karṣa/ suvarṇa ←16 ←māṣa ←5 guñjās/(10,5)

## COINS

From the *Arthaśāstra*:

1. *paṇa*, half *paṇa*, one quarter *paṇa*, and one eighth *paṇa*
2. *māṣaka*, half *māṣaka*, *kākaṇī*, half *kākaṇī*

From the Manusmṛti:

*kṛṣṇala, māṣaka, kārṣāpaṇa, śatamāna, suvarṇa, niṣka*

From the Līlāvātī/Kriyā:

*varāṭaka (cowry shells), kākiṇī, paṇa, dramma, niṣka*

<u>Seeds and grains mentioned in relation to silver and gold weights in the Arthaśāstra 2.19 (from the smallest)</u>	<u>Seeds and grains mentioned in relation to silver and gold weights in the Manusmṛti and Yājñavalkya</u>	<u>Seeds and grains mentioned in relation to silver and gold weights in the Līlāvātī and Kriyā</u>
MUSTARD SEED (gaurasarṣapa)	MUSTARD SEED (lower weight mentioned before the basic unit that forms the first denomination of weight)	YAVA ‘barley corn’
MĀṢAS-BEAN	YAVA ‘barley corn’ (lower weight mentioned before the basic unit that forms the first denomination of weight)	GUÑJĀ ‘seed of the Abrus Precatorius’
GUÑJĀ ‘seed of the Abrus Precatorius’	KṚṢṆALA ‘seed of the Abrus Precatorius’	VALLA ‘kind of wheat’
ŚIMBA ‘kidney bean’		
RICE- GRAIN		

## Measuring units and numbers in Old Babylonian Mesopotamia<sup>3</sup>

### I- Measuring units

**Units of capacities** (1 sila<sub>3</sub> ≈ 1 litre)

gur ←×5- bariga ←×6- ban<sub>2</sub> ←×10- sila<sub>3</sub> ←×60- gin<sub>2</sub> ←×180- še

**Units of weight** (1 gun<sub>2</sub> ≈ 30 kg)

gun<sub>2</sub> ←×60- ma-na ←×60- gin<sub>2</sub> ←×180- še

**Units of surfaces and volumes** (1 sar ≈ 36 m<sup>2</sup> / 18 m<sup>3</sup> / 720 bricks)

GAN<sub>2</sub> ←×100- sar ←×60- gin<sub>2</sub> ←×180- še

**Units of length** (1 ninda ≈ 6 m)

danna ←×30- UŠ ←×60- ninda ←×12- kuš<sub>3</sub> ←×30- šu-si

### Bridges

- between length and surfaces: 1 sar = 1 ninda × 1 ninda
- between surfaces and volumes:
  - 1 sar-volume = 1 sar × 1 kuš<sub>3</sub>;
  - 1 gin<sub>2</sub>-volume = 1 gin<sub>2</sub> × 1 kuš<sub>3</sub>, and so on;
- between volume and capacity: 1 sar-volume = 60 gur (that is, 1 gin<sub>2</sub>-volume = 1 gur, or, in other words, a cube of 6 šu-si edge has a capacity of 1 sila<sub>3</sub>).

### II- Numbers

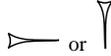
#### System G

Used for counting the highest unit of surfaces, namely, GAN<sub>2</sub>

	←	×10		←	×6		←	×10		←	×3		←	×6		←	×2	
šar'u			šar <sub>2</sub>			bur'u			bur <sub>3</sub>			eše <sub>3</sub>			iku			ubu
10 800			1 080			180			18			6			1			1/2

#### System S

Used for counting the highest unit of each system, except surfaces

	←	×10		←	×6		←	×10		←	×6		←	×10	
šar'u			šar <sub>2</sub>			geš'u			geš <sub>2</sub>			u			aš or diš
36 000			3 600			600			60			10			1

#### Sexagesimal Place Value Notation (SPVN)

... ← ×10 ← ×6 ← ×10 ← ×6 ← ×10 ← ×6 ← ×10 ← ...

<sup>3</sup> This presentation relies on metrological lists and tables used in OB scribal schools. The metrology used in other periods, or in other contexts, may be different. Moreover, the distinction between measurement units and numbers is not always as clear as suggested in this presentation.

## Basic measuring units in ancient China

The system of measuring units presented in the chapter entitled “Monograph on Pitch Pipes and the Calendar (*Lü li zhi* 律曆志),” in Ban Gu’s *History of the [Western] Han [Dynasty]* (end of the 1<sup>st</sup> century CE), denotes the system enacted by state institutions (Ban Gu 班固 et al. 1962:955—972). Note that, as far as we know, this system of measuring units was not the one used by Western Han institutions. When exactly this system was enacted is not completely clear. It is sure that, when Wang Mang 王莽 (45 BCE—23 CE), a reformer who attempted to overthrow the Han dynasty and establish a new dynastic rule, the Xin 新 dynasty (9—23), introduced new weights and measures, he used this system. The Xin dynasty was short-lived and the Han rule was reestablished in 25 CE (Eastern Han, 25—220 CE).

**LENGTHS.** The units of length measurement as described in the *History of the Han* form the following system:

**(1)** “The standards for measuring length are the *fen*, the *cun*, the *chi*, the *zhang*, the *yin*.

(...) Ten *fen* make a *cun*, ten *cun* make a *chi*, ten *chi* make a *zhang*, ten *zhang* make a *yin* and the five standards for measuring length are understood.

度者，分，寸，尺，丈，引也

(...)

十分為寸，十寸為尺，十尺為丈，十丈為引，而五度審矣。”

(Ban Gu 班固 et al.

1962:chapter 21a: 966).

The successive units have a decimal relationship.

This does not hold true for the *bu*, another measuring unit for length, which does not occur in the “Monograph on Pitch Pipes and the Calendar (*Lü li zhi* 律曆志).”

300 *bu*步 makes 1 *li* 里.

**System for volumes:** The measuring units *chi* and *cun* for lengths are also used to express volumes, by means of the following principle. A basic unit is chosen, for instance, the *chi*. The length “two *chi* seven *cun*” thus designates the height of a parallelepiped whose square basis has a side of one *chi* long. The same expression “two *chi* seven *cun*” also expresses a volume equal to that of this parallelepiped. In this system, whether *cun* represents a unit of length or a unit of volume, its relation to the unit *chi* is the same: 1 *chi* equals 10 *cun*. One could alternatively take the *cun* as the basic unit, the height expressed would then be that of a parallelepiped with a square basis of 1 *cun* side, and the relation between the *chi* and the *cun* would remain invariable. In that latter case, the same volume would be expressed by means of the quantity 2700 *cun*.

**System for surfaces:** The rectangular area whose width is 1 *bu* and length is *n bu* embodies the amount of area designated as *n bu*. The same construction holds true for *li*.

Specific units for surfaces are built upon these units, which are not mentioned in the “Monograph on Pitch Pipes and the Calendar (*Lü li zhi* 律曆志)”:

240 *bu* make 1 *mu* 畝.  
100 *mu* make 1 *qing* 頃

**CAPACITIES.** The system of measuring units for capacity, which the “Monograph on Pitch Pipes and the Calendar” of the *History of the Han* describes as follows, is also decimal:

(2) “The standards for measuring capacities are the *yue*, the *ge*, the *sheng*, the *dou*, the *hu*. (...) Coupling *yue*’s makes a *ge*, ten *ge* make a *sheng*, ten *sheng* make a *dou*, ten *dou* make a *hu* and the five standards for measuring capacities are excellent. 量者，龠，合，升，斗，斛也 (...) 合龠為合，十合為升，十升為斗，十斗為斛，而五量嘉矣。”

The units of capacity used in mathematical documents are in general taken from this list. However whereas some do designate the highest measuring unit as *hu*, some designate it as *dan* 石. The most frequently used units include the *hu* (*dan*), the *dou* and the *sheng*, the relationships of which can be represented as follows:

1 *hu* (*dan*) = 10 *dou* = 100 *sheng* (1 *sheng* was roughly speaking 0.2 liter)

**WEIGHTS.** In contrast to these two systems, the system for weight described is not decimal, as is clear from the following quotation of the same Monograph:

(3) « The standards for weighing are the *zhu*, the *liang*, the *jin*, the *jun*, the *dan*. (...) Twenty-four *zhu* make a *liang*, sixteen *liang* make a *jin*, thirty *jin* make a *jun*, four *jun* make a *dan*. 權者，銖，兩，斤，鈞，石也 (...) 二十四銖為兩，十六兩為斤，三十斤為鈞，四鈞為石。” (*History of the Han*, chapter 21a, p. 969).

24 *zhu* = 1 *liang*, 16 *liang* = 1 *jin*, 30 *jin* = 1 *jun*, 4 *jun* = 1 *dan*.