

SAW Three-Month Workshop:
Mathematical Practices in relation to the Astral Sciences

Abstract

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**THE PRODUCTION OF INTEGERS AND MATHEMATICAL OPERATIONS
IN WRITINGS ON ASTRAL SCIENCES IN ANCIENT CHINA**

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The earliest extant mathematical writings in Chinese, written between the end of the third century BCE and the fifth century CE, attest to a change of meaning of the term *chu* 除. In the oldest documents, whether they are manuscripts or writings handed down through the written tradition, *chu* refers to subtraction, or repeated subtraction. It is never used alone to refer to division. By contrast, in the extant documents that date from the first century or later, *chu* usually refers to division. When it does not, and there could be some ambiguity, for instance in the classic *The Nine Chapters on mathematical procedures*, commentators gloss the term to indicate that it means subtraction. In correlation with this change of terminology, one can argue that there was a shift in the execution of division and also in the theoretical part played by division (Chemla 2014). However, throughout this period, a fact remained unchanged: the result of a division was composed of an integer, or a sequence of integers with respect to a measurement unit, concluded by a fraction of the final measurement unit. The latter fraction was composed of a numerator and a denominator, and accordingly the result of the division was exact (Chemla 2013).

Selected calendrical systems were presented in technical treatises devoted to the topic, in dynastic histories composed between the 1st and the 7th century CE. They represent the main evidence we have about mathematical practices in the astral sciences at the time. The mathematical procedures, that according to the evidence of the dynastic histories the calendrical systems put into play, also used divisions and regularly the term *chu*. What can the observation of the type of quantities and key operations used in these procedures, or that of the terminology, tell us about the relationship between the mathematical practices to which the calendrical systems attest and those evidenced in the various mathematical writings from ancient China that survived? This was the key issue addressed in this communication.

The presentation brought to light several specific features of mathematical practices to which the procedures attest (e.g., specific types of quantities taken as operands or shaped as results of operations; in particular, specific nature of the result of divisions; specific operations on these results; in particular, specific operations on quantities of the type integer and fractions, when compared to those to which mathematical writings testify). These features, among others, seem to indicate that the practitioners who produced these procedure texts worked within the context of a culture of computation and quantification different from those evidenced by mathematical writings. This does not mean that there was no overlap between these cultures. The history of the meaning of the term *chu* suggests that information flowed between these various contexts. Indeed, in the earliest of these texts, from the first years of the common era, the term *chu* is used in ways wholly comparable with what we have in the manuscripts, while the prescription of division is carried out using more or less the same expressions. However, the term *chu* appears to refer to division in a calendrical system produced at the end of the first century, the “Quarter-remainder” system, showing echoes with mathematical writings arguably completed a few decades earlier. The use of *chu* as referring to division will become more and more prominent in later calendrical systems. On the other hand, if in the “Quarter-remainder” system, division is prescribed in the same way as in mathematical writings, the results of division remain specific to the tradition of the astral sciences and they are still the object of a specific arithmetical work. This feature illustrates a mixture of both overlaps and differences, which we will seek to describe in greater detail.

Bibliography

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