

SAW Conference: Mathematical Practices in relation to the Astral Sciences

March 26-27-28 and 30-31, 2015

The conference is organised by **Karine Chemla** (Sphère, CNRS & University Paris Diderot, SAW Project), **Matthieu Husson** (CNRS, SYRTE, Observatoire de Paris & SAW Project), **Agathe Keller** (Sphère, CNRS & University Paris Diderot, SAW Project), **Christine Proust** (Sphère, CNRS & University Paris Diderot, SAW Project) and the SAW group.

**Venue: Paris Diderot University, Condorcet Building
10 rue Alice Domon et Léonie Duquet 75013 Paris,
<http://sawerc.hypotheses.org/practical-information>**

Room 366 A

9:30 am-5:30 pm

I. Presentation

The conference “Mathematical Practices in relation to the astral sciences” is the final conference of the Phase 2 of the SAW Project. It follows upon the conference Cultures of computation and quantification in the ancient world, held two years ago, which concluded the Phase 1 (<http://sawerc.hypotheses.org/workshop-cultures-of-computation-and-quantification/conference-cultures-of-computation-and-quantification-in-the-ancient-world>).

The conference aims at describing specific “mathematical cultures” that took shape in the context of astral sciences. Moreover, it intends to highlight contrasts between cultures identified in these contexts and mathematical cultures to which other documents testify. We hope to go on with our exploration of “cultures of computation,” aiming at understanding better what

a “culture of computation” can be and which features are most relevant to perceive and characterize it. We also hope to widen our scope and explore how other facets of mathematical activities, e.g. geometrical practices, practice with instruments or practices of reasoning, carried out in the context of astral sciences could fruitfully lend themselves to a description in terms of “culture,” and highlight diversity inside and among mathematical cultures.

II. Programme

March 26, 2015

Quantities and Measurements Units

Chair: Clemency Montelle

9:30 am-11:00 am

Alexander Jones (New York University, USA)

Approximate results and recursive algorithms in Greek astronomy

Abstract: From the Hellenistic period on, Greek astronomy began to operate with precise numerical parameters, especially periodicities, in modelling the phenomena and motions of the heavenly bodies. A conflict between two approaches can be seen. In one, it was assumed that true values of parameters can be expressed exactly in terms of whole numbers, often reflecting numerological patterns. In the case of periodicities, such an assumption implies the existence of long combined recurrence periods encompassing multiple periodicities, e.g. for all the planets at once. In the other approach, all values of parameters are understood as approximations. Ptolemy's adherence to this view is closely bound with his extensive use of recursive deductive arguments in the *Almagest*.

Commentators: Sho Hirose and Daniel Morgan

11:00 am-11:30 am: Break

11:30 am-1 pm

Wang Xiaohu (School of Public Administration, South China Normal University, China)

Simulating Clepsydra: Two Methods of Time Measurement for Military in Ancient China

Abstract: Clepsydra (漏刻) was the most important time measurement device in ancient China. To measure time, Clepsydra required two basic components: Pots (壺) and Scale-rulers (漏箭). The Pots included Water-dropping Pot (泄水壺) and Water-receiving Pot (受水壺), which allowed the water to flow continuously just as time elapsed. The Scale-rulers were made of wood or bamboo, floating on the water in the Water-receiving Pot. When the clepsydra was working, people could evaluate the rising of the water level through the graduation on the Scale-ruler, thus the elapsed time could be quantified.

However, the clepsydra could not be used to measure time separately, and the operation was closely related to the official calendrical systems. When ancient Chinese used the clepsydra to measure time, a whole day was often divided into two parts: day time and night time. When night time ended, day time began; whilst when day time ended, night time began. Therefore there were two starting points for time measurement: sunrise and sunset. This operation was quite different from the clocks and watches we are using nowadays. Indeed, since the cycles of sunrise and sunset - the time starting points - changed annually, ancient Chinese had to change different scale-rulers to adapt it. A Chinese official calendrical system (曆法) was a special astronomical knowledge system that included series of data about different lengths of Days & Nights (晝夜長短) or different times of Sunrise and Sunset (日出入時刻). This is why the ancient Chinese changed scale-rulers of clepsydra.

Besides clepsydra, ancient Chinese also developed many methods for time measurement. In this paper, two procedures evidenced in military documents are introduced: Pace-counting (數步) and Bead-counting (數珠). While former researchers thought these methods were different 'instruments of measuring time' for the military in ancient China, this paper aims at demonstrating that they were the simulations of the clepsydra. And according to the data in the original texts, the two methods had to be operated with the length of days & nights, provided by the official calendrical systems.

The Pace-counting was recorded in *Taibai Yinjing* (太白陰經 *The Yin book for Venus to Overwhelm the enemy with strategies*, ca. 8th century), in the Tang dynasty, and many later military documents copied this text. It is assumed that the operator could walk 200 *li* (里) constantly in a whole day's time, which contained 100 *ke* (刻) in ancient China. Therefore, the operator's walking speed was 2 *li* per *ke*. Another military document of Ming dynasty, *Zhanshou Quanshu* (戰守全書 *Encyclopedia of attack and defense*, ca. 1628~1644) used the same method.

The Bead-counting was first mentioned in *Wujing Zongyao* (武經總要 *General outlines of different military canons*, 1044), from the Song dynasty, and recorded in *Jixiao Xinshu* (紀效新書 *New treatise on military efficiency*, 1560) from the Ming dynasty. It also required a

whole day's time to be 100 *ke*, and the operator shifted a string of beads at a uniform speed while walking. While one had been walking a pace, he shifted one of 740 beads of the string by his fingers. In one *ke*, he could shift a few more than 740 beads and walk as the same number of paces. So the operator would finger 74700 beads and walk 74700 paces in a whole day.

Pace-counting and Bead-counting were relatively simple and could be operated easily in the military. For the two methods to be used, starting points for time measurement, such as sunrise and sunset, were also required and they changed annually. *Taibai Yinjing* (太白陰經) offered the night length of 12 Minor Solar Terms (中氣 Zhongqi). In this paper we provide evidence showing that lots of data were from the Calendrical System Yuanjia (元嘉曆, 445) and the Calendrical System Daming (大明曆, 510). The data provided by *Zhanshou Quanshu* (戰守全書) for the night length at Spring Equinox, Summer Solstice, Autumnal Equinox, Winter Solstice were from the Calendrical System Datong (大統曆, 1385). The *Jixiao Xinshu* also mentioned the use of day & night time at different Solar Terms (節氣).

With the idea of simulating clepsydra, ancient Chinese tried to walk and shift beads at a uniform speed. In fact, the process of counting paces and beads was very similar to the quantification of continuous water flow between the Water-dropping Pot (泄水壺) and Water-receiving Pot (受水壺). While measuring time, the day & night time in the official calendrical systems, had to be taken into account. The principle behind this method, was the same as the changing Scale- rulers of clepsydra.

By looking at these cases we offer a new perspective on the understanding of ancient Chinese time measurement.

Commentators: Daniel Morgan and John Steele

1 pm-2:15 pm: Lunch Break

Progressions and ordered lists of numbers

Chair: Mathieu Ossendrijver

2:15 pm-3:45 pm

Michio Yano (Kyoto Sangyo University, Japan)

Geometrical model and observation in Indian astronomy

After the introduction of Hellenistic astrology and astronomy, the Indian counterparts underwent a drastic change. It is not yet clear exactly when this change started, but the first flow was the *Yavanātaka* which belonged to the 3rd or 4th century. In this text, the astrology of the Hellenistic origin is found. However, the elements of mathematical astronomy found in the last chapter (Chapter 79) are not Hellenistic, but rather similar to that of Mesopotamia. Āryabhaṭa was the first Indian astronomer who discussed mathematical astronomy of Greek origin using trigonometry and geometrical models. What is important is that the Greek astronomy transmitted to India was that of shortly before Ptolemy. Once the basic models were imported, Indian astronomers devised their own methods of obtaining planetary position. They used two epicycles separately, one *manda* epicycle to explain the eccentricity of planets, and another *śīghra* epicycle to deal with the anomaly of a planet's position relative to the sun. Instead of combining the two epicycles, they prepared two correction tables separately and tried to explain the two irregularities by algebraic manipulation and they tried to modify parameters and the sizes of epicycles. Some of these modifications should have resulted from observations. What I would like to discuss is the role of observation in Indian astronomy.

Commentators: Zhihui Chen and Christine Proust

3:45 pm-4 pm: Break

4 pm-5:30 pm

Steve Shnider (Bar Ilan University, Israel)

Difference schemes and their relation to interpolation rules in Babylonian lunar ephemerides

Abstract: Babylonian astronomical procedure texts give detailed instructions for interpolation in tables of lunar phenomena, but there is very little explanation of the underlying principles. Asger Aaboe's publication and analysis of the tablet BM 36311 provided a crucial insight into one of the major interpolation procedures in the System A lunar ephemerides. The tablet contained a large section of an astronomical table with 4 columns and 124 rows. There was no explanatory text, but the first two columns were easily recognized as truncated forms of Column *Phi* from the System A lunar ephemerides. While studying the structure of the table, Aaboe realized that the line to line increments in the third column were equal to the differences between the entries in the first two columns, which relation, expressed in a mathematical equation, becomes, $III(n+1) - III(n) = II(n) - I(n)$. We will discuss the source of this relation and properties of the tables it generates, using the examples in the tablets BM 36311 and BM 36699 as well as related fragments as studied by Aaboe and later by John Britton.

Commentators: Pierre Chaigneau and Karine Chemla

March 27, 2015

Progressions and ordered lists of numbers (continued)

Chair: Glenn Van Brummelen

9:30 am-11 am

Karine Chemla (Sphère, CNRS & University Paris Diderot, SAW Project) and Li Liang (Institute for the History of Natural Sciences, Chinese Academy of Sciences, China & SAW Project)

Progressions, motions and changes in the astral sciences of ancient China

Numerical progressions (hereafter, progressions) are an important topic in mathematical writings. They also occur in relation to astral sciences, in particular in the official monographs of dynastic histories which presented some calendrical systems that were used in ancient China. In this talk, we focus on the progressions dealt with in these two types of documents. A first point we make is that in mathematical writings, one finds two key types of problems related to progressions. In a first type, knowing the sum, or some of the terms, the problem asks to determine the progression, on the basis of assumptions with respect to its nature. In a second type, knowing the initial term and the common difference of an arithmetical progression, one determines its sum. Our second point is to emphasize that both problems have parallels in the astral sciences. In some cases, astronomical systems provide progressions to handle variations of specific data along time, in ways that are reminiscent of problems of the first type. Different actors appear to have shaped progressions according to wholly different principles, especially with respect to symmetry assumptions. In other cases, actors compute planetary motions along a cycle in ways that are reminiscent of the second type of problems. In brief, in the contexts of the writings we have on astronomical systems, we encounter terms and procedures that are common with mathematical writings whereas in mathematical writings, we encounter procedures that evidence the influence of terms and modes of thinking coming from the astral sciences. In the conclusion, we discuss hypotheses about the relationships between these two types of documents to which the topic of progressions attests. Moreover, our talk discusses a third point, namely the evidence available about criteria some actors in the astral sciences used to assess the types of progressions used by their predecessors. These pieces of evidence could be important to understand how actors thought one should combine observation and mathematics in opting for a given progression.

Commentators: Mathieu Ossendrijver and Martina Schneider

11 am-11:30 am: Break

Mathematical Reasonings 1

11.30 am-1pm

Sho Hirose (Sphère, CNRS & University Paris Diderot, SAW Project)

Explanations on planes, explanations in space: the usage of the armillary sphere in Parameśvara's Goladīpikā

The *Goladīpikā* (literally "Illumination of the spheres"), composed in two versions by the south Indian astronomer Parameśvara (c.1360-1460), starts with the description of an armillary sphere, which is an unusual feature compared to other Sanskrit sources that usually refer to the instrument in the middle or near the end of the treatise. However, in the following parts of the *Goladīpikā* the armillary sphere is never explicitly mentioned.

Some topics are likely to have been explained on a plane; in fact, in the case of the movement of the shadow of a gnomon, the parallax and the eclipses there are even instructions to draw diagrams on the ground.

Meanwhile, some topics involve multiple planes. For example, the calculation of declinations, planetary latitudes and heights of the sun involves Rules of Three, which come from similar triangles in different planes. The text not only gives the Rules of Three themselves but also the descriptions of the triangles related to them. Such explanation could be done either by a projection on a plane or with an armillary sphere.

Other topics like the ascensional difference occur on a curved surface. It is debatable whether this was explained by projecting on a flat plane or by an instrument. The aim of this talk is to see whether we can distinguish explanations in a plane or in space, and also to see why these various topics of seemingly different nature have been put together.

Commentators: Matthieu Husson and Nathan Sidoli

1 pm-2:15 pm: Lunch Break

Chair: Bill Mak

2.15 pm-3.45 pm

Seturo Ikeyama (Kyoto Sangyo University, Japan)

Practical Trigonometry in Indian Astronomy

Abstract: Trigonometry came out of the practice of astronomy and they are inseparably linked to each other. It is true in the case of Indian astronomy.

In my speech, I will introduce examples of trigonometrical calculation showing how Indian astronomers solved problems on the celestial sphere without using any techniques of spherical trigonometry, nor even the concept of angle. I will mainly speak of the calculation of "drk-ksepa", which corresponds to the zenith distance of the midpoint of the ecliptic above the horizon, and a subtle way for calculating the sum of two sines which are not on the same plane. Both of them are mentioned by an Indian astronomer, Nilakantha, in his commentary on *Aryabhatiya* written in the 14-15th century.

Commentators: Agathe Keller and Li Liang

3.45 pm-4 pm: Break

4 pm-5.30 pm

Chen Zhihui (Sphère, CNRS & University Paris Diderot, SAW Project)

How Did Mid-Qing (late 1790s) Scholars Use Ancient Calendrical Methods to Calculate Dates of Events Mentioned in Confucian Canons? -- A Study Focusing on Li Rui's (1768-1817) Zhaogao Riming Kao (Research on the Names of the Days in the Announcement by the Duke of Zhao, ca. 1800)

Abstract: In ancient China, the texts about mathematical practices occurred not only in the mathematical books but also in the commentaries on Confucian canons. And some research has been done on the characteristics of the different mathematical knowledge in Confucian Classics, such as the works on the commentary on the *Rites of Zhou* in the 7th century by Zhu Yiwen (朱一文).

For historians specialized in the Confucian Classics, finding the dates of historical events recorded in Confucian canons is one of the important issues. *Zhaogao* (召誥 *The Announcement by the Duke of Zhao*) and *Luogao* (洛誥 *The Announcement announced in the Capital Luo*) are two successive chapters in *Shangshu* (尚書 *Book of Documents*, one of the 'Five Classics' for Confucians, a collection of politics related documents in ancient China). Some months and sexagesimal stems and branches of the dates (*ri ganzhi* 日干支; or names of the days, *riming* 日名) without year when the events took place are clear in the *Zhaogao*, while it can be determined that the *Luogao* was announced in the seventh year of the King Cheng of the Zhou (ca. 11th BCE) according to the text in the *Luogao* itself. Zheng Xuan (鄭玄 127-200 CE, a famous Confucian scholar who commented on all kinds of Confucian canons) believed that these two *Announcements* were announced in the fifth year and the seventh year of the King Cheng, respectively. Accordingly, Zheng Xuan alleged that the text of the *Zhaogao* was not correctly transmitted, and 'the second month' and 'the third month' should have been 'the first month' and 'the second month' in the original text. Kong Yingda (孔穎達 574-648 CE,

who was in charge of compiling for the subcommentaries on the ‘Five Classics’ for Confucians) believed that the texts of the two *Announcements* were both shaped in the seventh year of the King Cheng, and that there were no mistakes on the months in the ancient editions. Kong Yingda also computed and proved it by a ‘mathematical method’, which was recorded in the subcommentary on the *Book of Documents*.

The mid-Qing scholars studied this topic further by using ancient astral-mathematical methods. At first, two classicists, Wang Mingsheng (王鳴盛 1722-1797) and Jiang Sheng (江聲 1721-1799) computed the total months between the fifth year and the seventh year of the King Cheng with the data in ‘the Six Ancient Calendars’ (*gu liuli* 古六曆), arguing against the opinion that the two *Announcements* had been written in the same year. And then, mathematician Li Rui (李銳 1768-1817) wrote the *Zhaogao rimingkao* (召誥日名考 *Research on the Names of the Days in the Announcement by the Duke of Zhao*, ca. 1800). Li Rui not only obtained a chronicle about the events in the two *Announcements*, but also he could suggest a daily table with the events that took place in the first and second month of the fifth year of the King Cheng using a reasoning and a computation grounded on methods, found in the Calendrical system named *Quarter-remainder* (*Sifen li* 四分曆, 85 CE). However, in order to make the result matched the conclusion by the ancient commentator Zheng Xuan, he adjusted the first year when the Duke of Zhou became a regent in his chronicle which could not be supported by other evidence. And, Li Rui’s work had a profound influence on the later mathematicians such as Luo Shilin (羅士琳 1789-1853).

Commentators: Clemency Montelle and Zhou Xiaohan

March 28, 2015

Mathematical Reasonings 2

Chair: Nathan Sidoli

9.30 am-11 am

Hermann Hunger (University of Vienna, Austria)

Reasoning involving mathematics in schematic Babylonian astronomy

Abstract: Numerical data on astronomical phenomena are of course present in early texts on star lore, but it is never stated how they were arrived at. It is therefore mostly guesswork

when we try to think about the mathematical reasoning used. In my paper, I shall present examples of mathematical practices that are unexpected - at least when compared to other cases which seem more reasonable to us.

Commentators: Magali Dessagnes and Matthieu Husson

11 am-11.30 am: Break

11.30 am-1 pm

Karine Chemla (Sphère, CNRS & University Paris Diderot, SAW Project)

Conjunctions between the sun and the moon, and pursuit problems. Mathematical reasoning in Chinese writings on astral sciences

The first problem of the early 7th century classic *Mathematical Canon continuing the Ancients* (緝古算經), by Wang Xiaotong, deals with a computation related to the determination of the new moon. In his own commentary on this problem, Wang Xiaotong develops a reasoning on the correctness of the procedure. In this context, he argues that his procedure is similar to the one that the 1st century classic *The Nine Chapters of mathematical procedures* describes to solve a problem. Arguably the problem is the 12th of chapter 6. Interestingly, commentators on *The Nine Chapters*, Liu Hui in the 3rd century and a group working under the supervision of Li Chunfeng in the 7th century, also develop reasonings to establish that the procedures given by the classic are correct. The commentary on problem 6.12 thus accounts for the correctness of the procedure given in *The Nine Chapters*. Furthermore, the commentary on the procedure given for problem 6.13 likewise argues that the reasoning underlying this procedure is the same as for 6.12.¹ The talk will examine these various reasonings, and the grounds on which the various actors claim identity or similarity between procedures. It will also examine what can be derived from different uses of the term 率 *lǜ* in these various documents.

In this problem, Wang Xiaotong claims to offer a “new” solution for a problem addressed in the official calendar. The method he criticizes for its cumbersome computations and suggests to replace by his own procedure, seems to be the one given in the *Wuyin yuan* calendrical system (戊寅元曆, enacted in 619), which Wang was appointed to examine in 623. Interestingly, only a few years later, in 627, Li Chunfeng is known to have criticized the same calendar and to have been accordingly appointed to the bureau of astronomy. These various elements will lead me to question what these documents tell us on the relationship between mathematical writings and writings on the astral sciences at the time.

¹ Wang Xiaotong’s problem, procedure and auto-commentary were for the first time for their greatest part translated into German and discussed in (Bréard 1999, 333-336, 41-43), respectively. (Bréard 2002) evokes the relationship between the problem from *The Nine Chapters* quoted and this problem by Wang Xiaotong as well as the relationship between pursuit problems and astronomical problems in other Chinese, Sanskrit and Latin sources. The overview given yields a very good starting point for an analysis of the relationship between pursuit and conjunction problems.

Bibliography

Bréard, Andrea. 1999. *Re-Kreation eines mathematischen Konzeptes im chinesischen Diskurs: "Reihen" vom 1. bis zum 19. Jahrhundert*. Stuttgart: Franz Steiner Verlag.

Bréard, Andrea. 2002. "Problems of pursuit: Recreational mathematics or astronomy." In *From China to Paris: 2000 Years of Mathematical Transmission*, edited by Y. Dold-Samplonius, J. W. Dauben, M. Folkerts and B. van Dalen, 57-86. Stuttgart: Franz Steiner Verlag.

Commentators: Baptiste Mèlès and Clemency Montelle

1 pm-2:15 pm: Lunch Break

Chair: Agathe Keller

2:15 pm-3:45 pm

Glenn Van Brummelen (Quest University, Canada)

Al-Kashi's Two Methods of Determining $\sin(1 \text{ degree})$: Finding Roots and Changing Paths

Abstract: Jamshid al-Kashi, the great 15th-century Persian astronomer and calculator, is well known for solving the problem of finding $\sin(1 \text{ degree})$ late in his career using an ingenious iterative technique. His method inspired several alternate, but essentially similar approaches over the following decades. Less known is his first attempt to find $\sin(1 \text{ degree})$ in the *Khaqani Zij*, which reflects more traditional methodological choices. We explore the mathematical and historical roots of both methods: what were the key features of his first method, where were his sources of inspiration, and how would his colleagues have reacted? Why might he have been dissatisfied with his first attempt, where might he have sought inspiration for the second method, and how might it have been received by his contemporaries and successors? The answers to these questions reflect on the changing nature of mathematics in general, and trigonometry in particular, in the early 15th century.

Commentators: Pierre Chaigneau and Sho Hirose

3:45 pm-4 pm: Break

4 pm-5:30 pm

Clemency Montelle (University of Canterbury, New Zealand, & SAW Project)

*What can numerical data reveal about approximation? Trigonometry and the associated interpolation rules in Brahmagupta's *Khaṇḍakhādyaka**

Abstract: In his *Khaṇḍakhādya*, Brahmagupta presents four rules which concern the determination of the sine and the inverse procedure of retrieving the arc. One of these verses includes his six tabulated sines-differences with a rule for linear interpolation to find a non-tabulated value. Another of these gives a second order interpolation procedure for determining non-tabulated sines. In a related manner, another verse gives a rule to determine the arc given the sine via a linear procedure, and the last gives a rule to find the arc from a given sine using an iterative procedure. The variety of rules and their placement in the text raise many pertinent questions. When should a linear approach be used over a non-linear one? Which ones has Brahmagupta used and when? What does the existence of more accurate alternative rules reveal about how these procedures and the data they generated were regarded by those that used the texts? In order to understand these issues, I survey all places in the *Khaṇḍakhādya* where Brahmagupta presents versified numerical data to detect where possible the procedures used to obtain the values. Furthermore, I analyze the twelfth century commentary of Āmarāja on these verses and the numerical derivations he includes to appreciate the rules and procedures used to generate the numerical data. Āmarāja's reflections on the precision of the results he generates reveal an ever increasing theoretical approach to approximation.

Commentators: Li Liang and Christine Proust

March 30, 2015

Mathematical Practices: Contrasts

Chair: Matthieu Husson

9:30 am-11 am

Mathieu Ossendrijver (Humboldt University, Germany & SAW Project)

Mathematical Practices in Late Babylonian Mathematical Astronomy

Abstract: The Late Babylonian corpora of mathematical astronomy and mathematics (450-50 BCE) share a number of common innovations, for instance regarding their terminology and their love for computations involving many-digit numbers. However, both corpora also exhibit significant differences, in particular an apparent mismatch between the mathematical tables that are extant and the ones that are actually needed for computing the astronomical tables. In this presentation I discuss agreements and differences between the mathematical practices

that underlie Babylonian mathematical astronomy on the one hand and the mathematical texts on the other hand.

Commentators: Li Liang and Martina Schneider

11 am-11.30 am: Break

11.30 am-1 pm

Nathan Sidoli (Waseda University, Japan & SAW Project)

Geometrical analysis and trigonometry in Greco-Roman mathematics

Abstract: In this talk, I discuss the ways in which theorems of Euclid's *Data* were used by Theon of Smyrna and Ptolemy in their treatments of Hipparchus' solar model.

Commentators: Karine Chemla and Clemency Montelle

1pm-2:15 pm: Lunch Break

Chair: Christine Proust

2.15 pm-3.45 pm

Daniel Morgan (Sphère, CNRS & University Paris Diderot, SAW Project)

*By Process of Elimination: Further Remarks on the Operation *chu* 除 in Early Imperial Mathematical Astronomy*

Abstract: This paper will return to the operation *chu* 除 (“eliminate”) as explored by Karine Chemla and myself on January 23, refining the terminology deployed, providing usage statistics, and exploring actors’ rare words of definition and expository. We will revisit how *chu* “eliminate” is used to prescribe (1) subtraction for a positive integer difference, (2) repeated subtraction of a single value for a positive integer remainder, i.e. *modulo*, (3) subtraction of a series of values for a positive integer difference/remainder, and/or (4) division for a positive integer quotient and remainder. We will examine in what contexts each meaning appears and quantify the change in the term’s deployment over time. We will also examine how parallel expressions of all four operations function differently and/or share *chu*’s semantic load, and how the preferences for these expressions change over time. The goal of this analysis is to provide positive, quantitative evidence of synchronic and diachronic diversity in the language of basic mathematical operations in first-millennium Chinese sources for mathematics and mathematical astronomy.

Commentators: John Steele and Zhou Xiaohan

3:45 pm-4 pm: Break

4 pm-5:30 pm

Agathe Keller (Sphère, CNRS & University Paris Diderot, SAW Project)

Understanding the vikala, Pṛthūdaka's interpretation of Brahmagupta's rules on quantities with sexagesimal parts

Abstract: In the mathematical chapter of the *Brāhmasphuṭassidhānta* (628), two rules evoke computations dealing with quantities which have sexagesimal parts. These rules are unique for what we know of Sanskrit mathematical texts. Pṛthūdaka (ca. 860), a commentator of Brahmagupta, provides his own peculiar interpretation of these rules that we will try to follow, providing a first-hand edition and translation of his commentary on them. He interprets the first rule, BSS.12.62, as an approximation of a square and he understands the second one, BSS.12.64-65, as a rule to approximate a square-root (of a difference or sum of squares including a quantity with a sexagesimal part). The way quantities with sexagesimal parts were shaped and operated upon will be discussed, slightly revising past interpretations concerning them. Along the way, how divisions were carried out, and how such quantities were used in a more astronomical context will be evoked.

Commentators: Zhihui Chen and Charlotte de Varent

March 31, 2015

Mathematical Practices: Synthesis

Chair: Karine Chemla

9.30 am-11 am

Matthieu Husson (CNRS, SYRTE, Observatoire de Paris & SAW Project)

Operations on sexagesimal numbers outside and inside astronomical texts: a comparison

Abstract: Between 1320 and 1330 in Paris, a handful of astronomers shaped the Parisian Alfonsine Tables which were used in Europe as the main astronomical computational tool for the next two centuries. An important part of their work focused on the numbers used in astronomical computations, their relation to measurements and quantities and the procedures ap-

plied to them in order to perform fundamental arithmetical operations. Several writings bear witness to these reflections: some of them are directly included in the Canons of astronomical tables, while others are presented in a more mathematical genre of texts called *Algorismus*. For the former we are going to consider John of Murs's *Tabule permanentes* and John of Lignièrès's *Tabule magne*, and for the latter John of Lignièrès's *Algorismus minutiarum* and John of Murs's *Tabule magne*. By analysing the distribution of topics between these documents and contrasting the presentation they display, we want to address two questions: what relations can we describe, from these documents, between the type of numbers and quantities found in astral sciences and those found in mathematics? How do actors work with an inherited number system and transform it?

Commentators: Mathieu Ossendrijver and Charlotte de Varent

11 am-11:30 am: Break

11:30 am-1 pm

John Steele (Brown University, USA & SAW Project)

Babylonian Shadow-Length Schemes: Between Mathematics and Astronomy

Abstract: Several cuneiform texts present schemes for the variation in the length of the shadow cast by a gnomon over the course of the day for the different months of the year (these texts are all discussed in J. M. Steele, “Shadow-Length Schemes in Babylonian Astronomy”, *SCIAMVS* 14 (2013), 3–39). These schemes combine aspects of traditional Babylonian astronomy such as a model for the variation in the length of daylight over the course of the year using a linear zigzag function with mathematical methods such as the use of reciprocal functions. Some of the texts presenting these schemes employ terminology which is mainly known from mathematical texts rather than astronomical texts and one tablet combines information on shadow lengths with a compilation of metrological material. In this paper I will discuss the place of the shadow-length schemes between mathematics and astronomy in Babylonia.

Commentators: Magali Dessagnes and Nathan Sidoli

1 pm-2:15 pm: Lunch Break

Chair: Setsuro Ikeyama

2:15 pm-3:45 pm

Zhu Yiwen (Sun Yatsen University, China)

Uncovering Astronomical Procedures Using Counting-diagrams: The Case of Qin Jiushao's Writing on Chinese Remainder Theorem

Abstract: The Chinese remainder theorem, originally called *Dayan* 大衍 procedure, was first written down by a thirteenth century scholar, Qin Jiushao 秦九韶 [1208-ca.1261 CE] in the *Mathematical Book in Nine Chapters* [*Shushu Jiuzhang* 數書九章, 1247 CE]. Based on a seventeenth century manuscript, transcribed by Zhao Qimei's 趙琦美 [1563-1624 CE] in 1617 CE, this paper aims to rethink the composition of a text for the procedure involving the use of counting-diagrams. Firstly, *Fangcheng* 方程 procedure is analysed. It is argued that there was another use of *Fangcheng* in the context of the astral sciences which differed from the one known through mathematical documents. Secondly, Qin Jiushao's *Dayan* procedure is further studied in two ways: a, the connection Qin establishes between *Fangcheng* and *Dayan* procedures is examined; b, Qin's use of counting-diagrams in order to uncover astronomical procedures is analysed. Finally, Qin Jiushao's personal life is considered as providing appropriate evidence for revealing Qin's effort to write on the Chinese remainder theorem within distinct cultures of computations respectively in mathematics and astronomy. In conclusion, the Chinese remainder theorem can be understood as the textualization of counting rods procedures.

Commentators: Agathe Keller and Baptiste Mèlès

3:45 pm-4 pm: Break

4 pm-5:30 pm

Taro Mimura (University of Manchester, UK)

The Formation of Zījs in the Integration of Indian and Greek Mathematical Tools

Abstract: Scholars in the medieval Islamic world received and reformed the Ptolemaic astronomical system found in the *Almagest*, and explored a physical cosmology by combining the Ptolemaic models with Aristotelian natural philosophy. As a result, they produced a distinctive genre of astronomical research on cosmology called *'ilm al-hay'a*, which was canonicalized by Naṣīr Dīn Ṭūsī (1201-1274) in *Tadhkira fī 'ilm al-hay'a*. On the other hand, they developed quantitative study of astronomy by composing *zījs*, which are astronomical handbooks containing tables and explanation for solving specific problems. Clearly, their tables were based on Ptolemy's *Handy Tables*; however, we must remark that their explanation part could not be based on the *Almagest*; for in the *Almagest* Ptolemy presented quite a few propositions with geometrical demonstrations to explain astronomical phenomena and concluded with compiling tables, but he never described solutions for solving particular problems. By

reviewing the history of astronomy in the medieval Islamic world, we are reminded that the earliest *zīj*s emerged as translations of Indian astronomical handbooks called *Siddhāntas* which consist of verses giving the processes of how to solve specific astronomical problems.

In this paper, I will elucidate how the *zīj*s were formed in the integration of the mathematical tools developed in Indian astronomy and those in Greek astronomy, by comparing the solutions to a similar astronomical problem in *zīj*s, the *Siddhāntas* and the *Almagest*. Moreover, I will point out that this integration produced a new mathematical tool, which was used for updating the *Almagest* around the time of Ṭūsī.

Commentators: Guillaume Loizelet and Emilie Villey