

A brief summary of the achievements of the SAW project

The general objective of the SAW project was to develop historiographic tools to perceive diversity in the mathematical practices and bodies of knowledge to which the available Chinese, Sanskrit and cuneiform sources from the ancient world attest. To achieve this aim, we have first focused on mathematical practices that can be documented in relation to administrative and economic activities (Phase 1). We have then explored mathematical activities evidenced in the context of astral sciences of the ancient world (Phase 2). Finally, we have dealt with the history of historiographies of ancient mathematics in the 19th and 20th century worldwide (Phase 3). In parallel with detailed case studies, we have developed more methodological and historiographic reflections, in particular on the notions of “practice” and “culture”, now widely used in the history and philosophy of science.

Some key questions guided our inquiry in administrative and economic activities, and also on the astral sciences: Can we establish relationships between the sources testifying to these activities and extant mathematical writings produced in the three geographical areas on which we concentrate? Or else, are the mathematical practices to which these activities attest the sign of independent mathematical developments? How can we detect differences between practices or between bodies of mathematical knowledge? Some of the results obtained on these issues are presented in three forthcoming books: for Phase 1, Mathematics and administration in the ancient world and Cultures of computation and quantification in the ancient world, and for Phase 2, Mathematical practices in relation to the astral sciences.

For Phase 1, in each geographical area, specific sources proved of special interest. For early imperial China, Peng Hao has established that administrative regulation texts and the earliest extant mathematical sources evidence tight relationships, indicating a close link between these mathematical writings and the state bureaucracy. In this context, thanks to the evidence provided by mathematical writings, Chemla and MA Biao could offer a new interpretation of a regulation for the state’s grain management, as quoted in a mathematical book. For the Indian subcontinent and Mesopotamia, new corpora to study mathematical practices in the context of economic activities were uncovered and explored. For Sanskrit, the sastras in general and Kautilya’s Arthashastra (ca 100 AD) in particular proved invaluable. Historiography had tried to make of this set of texts a testimony of a homogenous sanskritized metrology. Petrocchi’s research has shown the problems attached to this assumption. Middeke-Conlin’s PhD thesis (defended in June 2015) also established a corpus of economic documents from Southern Babylonia, and even more precisely Larsa in the early second millennium BCE, enabling him to initiate the study of mathematical activities in different administrations beyond studies of metrology. Middeke-Conlin could rely on these sources to reconstruct mathematical knowledge and practices at different levels of the bureaucracy, and he was able to highlight that at different hierarchical levels, officials in fact possessed different mathematical skills. He has suggested the hypothesis that this differentiation between different types of mathematical practices could be connected with the fact that different practitioners had followed different curricula of learning, whether they be academic or professional. Middeke-Conlin and Proust also explored the mathematical knowledge involved in economic transactions, through a study of a rare mathematical tablet (YBC 4698) providing a large overview of the mathematical approach to economic problems in the Old Babylonian period.

The study of mathematical practices in administrative contexts led us to focus more specifically on the shaping of numbers and quantities, and the means designed to operate on them. Michel researched Assyrian merchants’ mathematical practices at the beginning of the 2nd millennium BCE, bringing to light a specific

culture of computation and quantification, especially in relation to weighing practices. Proust researched the quantification of surfaces in Early Dynastic tablets from southern Mesopotamia considered the earliest mathematical tablets known, while Lecompte examined a set of fifty land surveyors' records dated from the same period and found in the same region. Zou Dahai and Chen Wei undertook similar exploration for China, through a study of the 6th century Mathematical Classic of the five bureaus, and brought to light a culture of computation different from what other mathematical writings evidence in China. We also devoted research to the quantification of spatial extension, and described how actors in the ancient world used for this either volumes or capacities, whether, depending on the context, they computed or measured them. Proust has also studied how various cultures of computation in Mesopotamia used metrological tables to establish relationships between volume, capacity and bricks units and quantities. Capacity units and vessels also proved essential topics for our purpose.

Studying computations on vessels in ancient China, Zhu has uncovered an entirely new corpus for the study of mathematical practices in ancient China: the 7th century commentaries on Confucian classics. He has shown that the mathematical practice they evidence is stable and of a nature strikingly different from what at the same time Li Chunfeng and his colleagues were gathering and annotating in the Ten Mathematical Canons. The work done in Phase 1 has thus allowed us to identify several distinct but overlapping mathematical cultures in ancient China. Chen Zhihui showed that at the end of the 18th century, two among these mathematical cultures could still be evidenced, and that actors perceived their difference.

Our research has more generally brought to the fore various ways of working with numbers/quantities and carrying out basic arithmetical operations on them. The new corpus discovered by Zhu testifies to a systematic use of a geometrical interpretation of quantities and operations on them to carry out computations, which differs from what mathematical canons in China show. On the other hand, by contrast to what this new corpus and also the early mathematical manuscripts evidence, Chemla has shown that the cultures to which mathematical canons attest is characterized by a specific use of positions in mathematical practice. This facet of the cultures of computation echoes Keller's research on some Sanskrit sources, showing a wide spread of diverse computations with positions, not restricted to the use of the decimal place value notation. A way of putting into play geometrical facets of quantities also appeared in the completely different cuneiform corpuses studied by Proust and Lecompte. In these contexts, like in others, it has proved essential that practitioners did not always compute with "abstract" numbers. The way in which actors operated on measurement values, and the nature of the "abstract" numbers occurring in computations, appeared a fruitful venue for research. Moreover, still in these cuneiform contexts, we put forward the hypothesis of a certain "granularity of quantities": the tools to compute with sexagesimal numbers, namely the metrological tables, might have embodied elementary pieces of quantities into which actors decomposed a given quantity for computation. This yields an entirely new insight into the perception of quantities, which sheds an unexpected light on how rounding was carried out in these contexts. This suggestion further raises the general issue of finding evidence testifying to actors' understanding of the quantities with which they operated. Another key fact related to quantities and computation was discovered: computing with small quantities might have been a different task from computing with large ones.

Finally, place-value number systems occurred in the three geographical areas under study, and our work has shed light on the differences between them and also between various milieus using them in each of these geographical areas, suggesting hypotheses on the processes of emergence, as well as on the use, of these systems. Notable in this respect is the study by Ouyang and Proust on marginal numbers found in some Ur III administrative texts. More generally, in her study on surfaces, Proust inquires into the circumstances of the invention of the sexagesimal place value notation. In relation to the issue of place-value number systems, arithmetical operations have appeared as essential objects of research for two reasons. Much variety can be evidenced in their execution, which reveals actors' work on operations. Moreover, the execution of operations sheds light on the variety of numbers and quantities with which actors dealt. With respect to Sanskrit sources, Keller and Singh have studied testimonies on operations in medieval Sanskrit sources, showing in particular variations in the execution of multiplication on integers. A striking result is that, contrary to a widespread assumption, once place-value decimal system was available, other ways of computing with numbers were still used. The same conclusion emerges from Zhu's research, which suggests that numbers were not approached in a uniform way throughout China. In another respect, Chemla established that the operation of division displays disunity between Chinese sources and also that a focus on division allows historians to understand that an important theoretical work has been devoted to that operation in early imperial China. The results of this theoretical work on division can be perceived in the shaping, in ca the first century CE, of both a mathematical knowledge about operations and also of a specific mathematical practice in which positions in general, and specifically positions attached to a place-value decimal system, played a key part. In other words, Chemla suggests that the emergence of a new execution of division might have been tightly connected with the emergence of a decimal place-value system. Her conclusions led Chemla to postulate the existence of two cultures of computation in early imperial China (one related to the recently found manuscripts from early imperial China, and another one related to most mathematical canons).

All the mathematical cultures that SAW described for ancient China, except the one discovered by Zhu Yiwen, are to a greater or a lesser extent documented in the Ten Mathematical Canons, an annotated anthology of old books edited under Li Chunfeng's (602-670) supervision, at about the same time when commentaries on Confucian canons studied by Zhu were composed. Li has played a major role in editing all the earlier Chinese mathematical writings that have come down to us. He was also the author of several technical treatises in a major historiographic work, the Book of Sui, as well as the author of a major calendrical system. Li Chunfeng was a poorly understood figure despite the numerous important roles that he played in the history of mathematics and astral sciences in China. Under the title Monographs in Tang official historiography. Perspectives from the technical treatises of the Book of Sui, SAW members Morgan, Chaussende and Chemla are completing the edition of a book devoted to the writing of technical treatises for the Book of Sui, which aims at studying Li Chunfeng's technical treatises in context.

Errors or mistakes appeared a key tool to help us perceive specificities of mathematical practices, and differences between them. Middeke-Conlin chose this angle to approach mathematical activities in the context of scribal schools and economic activities. He outlined a conceptual apparatus to distinguish among kinds of errors, which allowed him to show that approximation was a good indicator of mathematical cultures. Likewise, Chemla and Morgan began a cooperation on the nature of the mistakes present in the second-century BCE mathematical manuscript Suan shu shu. This research led them to identify a handful of phenomena that will not only change the way in which this text is read, but will shed light on features of early Chinese manuscript cultures beyond the sphere of technical literature. In particular, this cooperation inspired Morgan the project of carrying out handwriting recognition on all the manuscripts of the tomb, in which the Suan shu shu had been found (tomb 247 at Zhangjiashan, Hubei). Morgan's handwriting analysis of the entire tomb corpus led him to identify a network of hands, one of which appears across different manuscripts. Morgan could then establish the hand with which the tomb occupant was associated—this is one of the hands that occur in the Suan shu shu.

In the context of Phase 2—the study of mathematical practices in relation to astral sciences—the SAW project also made new corpuses of sources available. Ph. D. candidate Hirose's dissertation project bore on the Goladipika, a Sanskrit astronomical text devoted to spherical astronomy and composed by Indian astronomer Paramesvara around 1430-1450, and more precisely on its two versions (GD1 and GD2). Hirose completes a critical edition for GD2, with an annotated English translation, focusing in particular on the mathematical parts. In addition, Proust and Steele produced a critical edition of cuneiform astronomical and mathematical tablets from Hellenistic Uruk that were kept at the archaeological museum of Istanbul, with the intention of situating mathematical practices and astral sciences in the wider context of scholarship at Uruk. They also complete the edition of a book more generally devoted to scholarship at Uruk (Scholars and Scholarship in Late Babylonian Uruk). The post-doctoral scholarship awarded to Morgan enabled him to write a book (Astral Science in Early Imperial China: Observation, Sagehood and the Individual, forthcoming at Cambridge University Press) on the plurality of the astral sciences in early imperial China, including their mathematical facets. This book is an answer to monolithic modern narratives about 'Chinese thought' and 'Chinese science'.

Also in relation to the issue of documenting mathematical practices and their diversity, collective work in phase 2 of the project examined how writings in the astral sciences document mathematical practices and practitioners. A special issue, edited by M. Husson and R. Kremer, and devoted to this question has appeared in the journal Centaurus. It concentrates partly on various aspects of computation practice, and partly on uses of diagrams and instruments.

Collective work related to the book in preparation (Mathematical practices in relation to the astral sciences) has further focused on issues identified to be fruitful to perceive the diversity of mathematical cultures (in the context of the astral sciences or in relation of contiguity with them)—some of these issues are similar to some of the key issues investigated during Phase 1. These issues included: Operations, Quantities and Measurements Units, which manifest a diversity that has been overlooked owing to the widespread historiographic practice of converting numerical values and operations into modern counterpart (chapters by SAW members Chemla, Hirose, Keller, Morgan); Progressions and ordered lists of numbers, which proved to be essential and yet so far overlooked mathematical tools in ancient China and Mesopotamia, and could help connect sources in astral sciences and mathematical writings (chapters by SAW member Chemla, with Li Liang); Mathematical Reasonings, in the context of the astral sciences, and how they put space and time into play (chapters by SAW members Chemla, Hirose, Husson); Instruments, and the mathematical practices shaped to use them or work with them (chapters by SAW members Hirose, Keller, Husson, and articles by Morgan); Contrasts between and Syntheses of Mathematical Cultures (chapters by SAW members Husson and Zhu Yiwen).

The study of how mathematical practices and knowledge in mathematics and the astral sciences were put into play, or created, in relation to the need to conduct rituals in a correct way appeared to be a promising topic for the objectives of the SAW project, and a collective volume, edited by Hirose, Husson, Keller, and Morgan, is in preparation on the topic under the title Astral Sciences, Mathematics and Rituals.

Phase 3 had two facets. It included a theoretical reflection on mathematical (and more generally scientific) “practices” and “cultures”, and also a systematic exploration of the history of the historiography of mathematical sciences in the ancient worlds.

Theoretical work on “practices” and “cultures” focused on several dimensions of scientific work. We have devoted some attention to the study of epistemological values and their historiography (for instance, Chemla wrote two book chapters on abstraction). We have likewise conducted theoretical reflection on the variety of types of texts in the context of different scholarly cultures, and more generally on the issue of how documents reflect the milieus in which they were produced, and the mathematical practices that these milieus had. This is reflected in a forthcoming book, edited by Florence Bretelle-Establet and Stéphane Schmitt, titled *Parts and Pieces: an Exploration of the Textuality of Scientific Writings*, with chapters by several SAW members. Another outcome of our reflections is a set of articles, prepared under the responsibility of M.C. Bustamante, on the topic of Orality and the historian of science’s written sources, forthcoming in the *Archives internationales d’histoire des sciences*, with articles by Keller, Morgan, and Proust. In addition, we have collectively worked on critical approaches to the sources of ancient mathematics. One facet of this work dealt with the shaping of the sources on which the historians work. This included a research on the history of collections, in which historians find ancient mathematical sources. Such a concern is particularly vital for cuneiform tablets that were bought decades ago on the antiquities market, and it was the topic to which PhD student M. Dessagnes devoted her dissertation (defended in January 2017). From another perspective, we complete the edition of a collective book on the 19th and 20th history of critical editions and translations of ancient mathematical documents. In it, we place emphasis on the issue of the ancient editorial practices, which left an imprint on the documents and editions available, and we also pay due attention to the social contexts and circumstances that presided over the production of the modern editions under study. We analyze how key mathematical elements were dealt with in 19th and 20th century editions of ancient texts attesting to mathematical practice. We more generally focus on the making of early modern and modern editions of mathematical texts, approaching them in a contrastive way. In line with this issue, Chaigneau’s PhD thesis is devoted to Neugebauer’s and Thureau-Dangin’s critical editions of cuneiform mathematics in the 1930s and 1940s. The point is to examine critically how historiographic operations carried out in the editions of ancient texts influenced the historiography of mathematics in Mesopotamia. This last endeavor constitutes the second facet of our critical approach to the sources of the past. Our theoretical work on “cultures of mathematics” inspired the forthcoming introduction to K. Chemla & E. Fox Keller (eds.), *Cultures without culturalism*. The making of scientific knowledge, in which Chemla formulated suggestions about the actual historicity of scientific cultures, the circulation of specific concepts and practices between cultures, the correlation between cultures and concepts, and finally the historical making of universality of scientific knowledge. Chemla has also outlined her own way of approaching the diversity of mathematical culture in several articles, including a synthesis of her own work and also of other research works made by the SAW team, in her contribution to the Proceedings of the 7th Congress of the European Mathematical Society.

The second facet of Phase 3 included work on 19th and 20th century historiographies of ancient mathematics. To open this new fieldwork, it proved essential to pay attention to the diversity of milieus in the context of which, from the mid-18th century on, one can perceive an interest in the history of mathematics of the ancient world. This diversity enabled us to account for specificities of different historiographies, and also to describe how knowledge circulated and got transformed from one milieu into another. Major differences appeared in the history of historiographies of mathematics in the Indian subcontinent, China, and Mesopotamia. This is one of the main conclusions of the collective book in preparation under the title *Writing histories of ancient mathematics, 18th – 21st centuries*.

Sanskrit documents about mathematics and the astral sciences became available in English translations due to the work of scholars who had been based in India and were officials of the British crown or East India Company servants. Keller devoted much research to Colebrooke as a translator of the Sanskrit mathematical texts. With her help, Ivahn Smadja could devote several articles to the reception of Colebrooke’s translation by European mathematicians interested in ancient mathematics, in France and Germany. He established the interplay between the mathematics they were researching and how they read the ancient texts. He further showed how practices of interpretation of ancient mathematical texts widely differed. Olry Terquem appeared as a key figure in the introduction of the translations of Sanskrit mathematical texts into France as well as a key link between France and Germany. This latter facet relates to the fact that much later, Terquem also played an important part in the introduction of new information about the history of mathematics in China into Europe.

European studies of the history of mathematics in China followed entirely different paths. Chemla has worked on the sinologist trained in science Edouard Biot, and his early studies on the topic in 19th century Europe, which relied only on documents available in Europe. Post-doctoral scholar Chen Zhihui has done major research on the other main European author who wrote on the topic in the 19th century, protestant missionary Alexander Wylie, who was based in Shanghai, collected mathematical works on the Chinese book market, and was in working contacts with Chinese scholars versed in mathematics and the history of mathematics in China.

Chen has analyzed closely the much influential publication that Wylie devoted to the history of mathematics in China five years after his arrival in China, *Jottings on the science of the Chinese. Arithmetic*. He showed that Wylie heavily relied on knowledge gathered from the Penny cyclopedia (thereby suggesting how Wylie might have learned mathematics), and especially from mathematical entries mainly written by a key character for our research: Augustus De Morgan. Notes by Wylie on some mathematical books in Chinese allowed Chen to show that Wylie had in fact no complete command of the mathematical content. The collective book in preparation outlines a background both in China and in Europe for Wylie's breakthrough in the history of mathematics in China. Post-doctoral scholar Schneider and Chemla further analyze some specificities of Wylie's early historiography of mathematics in China—the one that was in fact influential in Europe. They suggest a reading of *Jottings* as closely connected with a reflection on the use of scientific knowledge for the protestant mission, and they provide evidence suggesting that the German translation of Wylie's article had political goals, aiming at convincing European powers to support the Taiping rebellion. This process of transmission, closely linked with missionary contexts and stakes, affected the information about mathematics in China, which in the end was lost, distorted, and sometimes partly reconstructed.

The book in preparation further inquires into the impact of De Morgan's mathematics and historiography of mathematics in India as well. It also examines several general issues that struck us as essential. First, some chapters focus on some of the key goals that spurred the development of historiographies of ancient mathematics in the 19th and 20th centuries. Second, the book develops a historical analysis of which mathematical knowledge has been used in the interpretation of ancient texts and which impact this had on the interpretation produced (chapters by Zhou Xiaohan, by Smadja, by Chaigneau and Baptiste Mèlès). Proust further focused on how historiography of ancient mathematics dealt with—and most often modernized and thus uniformized—numbers and quantities, especially in the work by David Eugene Smith. Keller likewise analyzes the various mathematical tools, conceptions of what a mathematical text should be, and wider narratives of history of mathematics that shaped different historians' reading of Sanskrit texts and their interpretation. Thirdly, our theoretical reflection on the history of historiographies of ancient mathematics has also borne on the history of the categories that informed the modern treatment of ancient sources. In particular, Hirose focused on the category of "schools of thought", and Morgan examined the use of some actors' category extended to all actors across social milieus and centuries. In the context of how mathematical categories have shaped the historiography of astral sciences, Hirose has underlined how the opposition between computational astronomy and one based on geometrical models has structured the questions asked to Sanskrit astral sources. Fourthly, the book examines how various actors made sense of the diversity of mathematics. This issue has inspired Christine Proust's work on David Eugene Smith and Martina Schneider's research on Moritz Cantor, and especially his book on the "cultural life of the peoples", approached from the viewpoint of the history of numbers. In line with this issue, we devoted special attention to the "cultural histories" of mathematics in the first decades of the 20th century (chapters by Mizuno and Chemla, and by Eric Vandendriessche).

This research has given tools to us to describe and situate more specifically some present-day uses of the history of ancient mathematics. For instance, Keller has pursued her work on "Vedic mathematics". Her analysis led her to conclude that "Vedic mathematics" testifies to the constant will, from the beginning of the 20th century until today, to create an alternative non-western and religious modernity, embodied more widely in the phenomena of "Vedic Sciences". A phenomenon not restricted to India, "Vedic mathematics" illustrates how science and its history have been, and are still used, to proclaim and define communities. It also shows how history of science is appropriated outside of academia.

SAW's results on cultures of computation presented above provided the foundations for de Varent's PhD thesis titled *Ancient mathematics in teaching: issues, practices and prospects (Les mathématiques anciennes dans l'enseignement : enjeux, pratiques actuelles et perspectives)*. Working at the boarder between math education and the history of mathematics, she explored how to use ancient mathematical practices and bodies of knowledge on numbers/quantities and arithmetical operations (mainly multiplication and division), as described and analyzed in the context of SAW, in present-day mathematics education. The idea is to show students the variety of past mathematical approaches to some of the topics taught, as an aid to understand these topics better and also as a contribution to making students experience concretely the international character of mathematics and its presence in many milieus of any given society. In relation to her work, several SAW team members, including Proust, Dessagnes, de Varent, Husson and Singh, explored a new direction of research with a historical approach to the introduction of history of mathematics in mathematics education in the 19th and early 20th century. The case of D. E. Smith (1860 – 1944) and his role in the promotion of both history of mathematics and mathematics education was central to this inquiry.

Last, but not least, the research conducted in the context of the SAW project enabled some members to offer a renewed vision of their field, which is reflected in articles and chapters of books that they composed for more general audiences.