



SPHERE

Laboratoire de Philosophie et d'Histoire des Sciences UMR 72191



Conference Practices of Reasoning in the Mathematical Sciences – 2017

organised by C. Proust, M. Husson, A. Keller and K. Chemla
in the context of the Seminar “History and Philosophy of Mathematics”

Venue: **Université Paris Diderot**, Condorcet Building, 10 rue Alice Domon et Léonie
Duquet 75013 Paris - **4-7 December, 2017 – 9:30 am to 5:30 pm - room 366**

Presentation

As part of the Seminar “History and Philosophy of Mathematics” of the SPHERE Laboratory, a four-day workshop on “Practices of Reasoning in the Mathematical Sciences” will be held on December 4-7, 2017 in Paris (University Paris Diderot). The discussion will be a continuation of researches on the diversity of mathematical reasoning, initiated in the framework of the SAW Project (2011-2016). Its goal is to describe a variety of practices of mathematical reasoning in a wide range of mathematical, astronomical and administrative documents, from diverse provenances and time periods. The participants will analyze the discursive components of reasoning, like terminology, as well as non-discursive elements such as diagrams, spatial organizations, tabular formats or notations. The workshop aims to prepare a collective book.

Speakers

CHEMLA, Karine (*SPHERE – CNRS & Université Paris Diderot, France*)

CHEN Jiangping Jeff (*Saint Cloud State University, MN, USA*)

CROZET, Pascal (*SPHERE – CNRS & Université Paris Diderot, France*)

HUSSON, Matthieu (*CNRS, SYRTE-Observatoire de Paris, France*)

KELLER, Agathe (*SPHERE – CNRS & Université Paris Diderot, France*)

LI Liang (*IHNS, Chinese Academy of Sciences, Beijing, China*)

MIDDEKE-CONLIN, Robert (*Max Planck Institute for the History of Science, Berlin, Deutschland*)

PROUST, Christine (*SPHERE – CNRS & Université Paris Diderot, France*)

QU Anjing (*Northwest University, Xi'an, China*)

REYNAUD, Adeline (*SPHERE – CNRS & Université Paris Diderot, France*)

ZHANG Yijie (*Sun Yat-sen University, Guangdong, China*)

ZHENG Fanglei (*IHNS, Chinese Academy of Sciences, Beijing, China*)

ZHOU Xiaohan (*SPHERE – CNRS & Université Paris Diderot, France*)

ZHU Yiwen (*Sun Yat-sen University, Guangdong, China*)

Programme

Monday, December 4th, 2017

Comparing modalities of reasoning in mathematical contexts and in other contexts

Chair: Karine Chemla

9:30 am – 1:00 pm – Room 366A-Klimt

Robert Middeke-Conlin (Max Planck Institute for the History of Science, Berlin, Deutschland)

Tabular administrative texts as a reflection of mathematical practice

Commentator: Matthieu Husson

Coffee break

Agathe Keller (SPHERE – CNRS & Université Paris Diderot, France)

Grounding and Explaining Sines: Pṛthūdaka's commentary on Brahmagupta's grounding of Sines in the Brāhmasphuṭasiddhānta

Commentator: Zheng Fanglei

Lunch

2:00 – 3:30 pm – Room 366A-Klimt

Qu Anjing (Northwest University, Xi'an, China)

Two cases of reasoning in Chinese mathematical astronomy

Commentator: Karine Chemla

4:00 – 5:30 pm

Harold Edwards (New York University, USA), exceptional lecture, in addition to the program

Galois's (Almost) Constructive Presentation of Galois Theory

Tuesday, December 5th, 2017

Role of diagrams and other artefacts in reasoning

Chair: Christine Proust

9:30 am – 1:00 pm – Room 454A-Valentin

Adeline Reynaud (SPHERE – CNRS & Université Paris Diderot, France)

What diagrams tell us about practices of mathematical reasoning in Old-Babylonian Mesopotamia: the example of the approximation procedure in tablet YBC 8633

Commentator: Agathe Keller

Coffee break

Matthieu Husson (CNRS, PSL-SYRTE-Observatoire de Paris, France)

Multifaceted mathematical reasoning in Jean des Murs' De moventibus et motis (1343)

Commentator: Christine Proust

Lunch

2:00 – 3:30 pm – Room 646A-Mondrian

Zhou Xiaohan (SPHERE – CNRS & Université Paris Diderot, France)

Methods using duan (段 segment [of diagram]) in Yang Hui's 楊輝 (fl. 13th century) works

Commentator: Adeline Reynaud

Wednesday, December 6th, 2017

Mixing cultures of reasoning

Chair: Matthieu Husson

9:30 am – 1:00 pm – Room 366A-Klimt

Chen Jiang-Ping Jeff (St Cloud State University, MN, USA and SPHERE)

Persuasion and Refutation in Fangcheng lun – A Case Study of the Practices of Reasoning in a 17th-century Chinese Algebraic Treatise

Commentator: David Rabouin

Coffee break

Li Liang (Beijing Institute for the History of Natural Science, Chinese Academy of Sciences, China)

The perceptions of Western and Islamic terminology for astronomical tables in early modern China.

Commentator: Robert Middeke-Conlin

Lunch

2:00 – 3:30 pm – Room 366A-Klimt

Pascal Crozet (SPHERE, Université Paris Diderot, France)

Abū Kāmil's algebraic proofs – La démonstration algébrique chez Abū Kāmil

Commentator: Jeff Chen

Thursday, December 7th, 2017

Meaning of steps and accounting for the correctness of procedures

Chair: Agathe Keller

9:30 am – 1:00 pm – Room 366A-Klimt

Christine Proust (SPHERE – CNRS & Université Paris Diderot, France)

Reasoning running through a series of problems. An analysis of some procedure texts from Mesopotamia, early second millennium BCE

Commentator: Pascal Crozet

Coffee break

Zheng Fanglei (Fudan University, Shanghai, China)

Reasoning with the participation of unknowns before they are found-demonstrative texts in Al-Khwarizmi's Algebra and Nemorarius' De numeris datis

Commentator: Matthieu Husson

Lunch

2:00 – 3:30 pm – Room 366A-Klimt

Karine Chemla (SPHERE – CNRS & Université Paris Diderot, France) & ZHU Yiwen (Sun Yat-sen University, Guangzhou, China)

Algorithms carrying out derivations (tui) versus Algorithms for looking for (qiu). On the first entry of Mathematical Procedures for the Five Canons

Commentator: A.J. Misra

Abstracts (by session)

Monday, December 4th, 2017

Comparing modalities of reasoning in mathematical contexts and in other contexts

Chair: Karine Chemla

Middeke-Conlin, Robert

Tabular administrative texts as a reflection of mathematical practice

Abstract – By the Old Babylonian period, that is, the early second millennium BCE in Southern Iraq, some administrations begin to produce texts in a tabular format, instead of a more common prosaic format. This record keeping practice, which is sporadically witnessed since the Early Dynastic Period in Mesopotamia (early- to mid-third millennium BCE), offers a significant, although temporary, improvement in data presentation. At the same time, numerous Old Babylonian mathematical texts take on a tabular format as well. Can any link be made between these two distinct varieties of texts?

This paper attempts to answer this question by examining both tables and tabular lists in the administrative record, as well as mathematical texts that take on or present a tabular format. It asks, “how did the tabular texts from the Old Babylonian mathematical tradition portray mathematical reasoning?”, “do the Old Babylonian tabular economic texts maintain a tabular format to express mathematical reasoning?”, and finally, “is mathematical reasoning in the economic texts expressed in a similar manner as in the mathematical texts?” knowledge.

Keller, Agathe

Grounding and Explaining Sines: Pṛthūdaka’s commentary on Brahmagupta’s grounding of Sines in the Brāhmasphuṭasiddhānta

Abstract – The aim of this paper, beyond its study of a specific case, is to raise a certain number of questions which spring from the existence of reasonings and modes of justification in Sanskrit mathematical and astral sources. The first and most essential being: are modes of justification different in mathematics and astral science? In the following, a famous reasoning found in Sanskrit literature -the one used to ground tabulated values of Sines and Sine differences- will be studied. Paradoxically, if such reasonings are often mentioned in the literature on the history of trigonometry, they rarely have been studied for what they obviously are, a kind of ‘proof’, in the sense that they try to provide a grounding and justification of why such values are used. This paper focusses on the explanations (*vāsanā*) Pṛthūdaka (ca. 870) provides while commenting on verses 17-19 of the Chapter on the Sphere (*golādhyāya*, Chapter 21) in Brahmagupta’s *Brāhmasphuṭasiddhānta* (628). In these verses, Brahmagupta returns to the values he has given earlier for Sines, and provides rules to justify each value, one by one. Pṛthūdaka’s commentary not only explicits these justifications but further provides what he calls ‘explanations’ for them. This way of ‘explaining’ Brahmagupta’s reasoning will be studied and characterised before being compared with the kind of ‘explanations’ (bearing the same name), that Pṛthūdaka provides while commenting on Brahmagupta’s mathematical rules, as they appear in the mathematical chapter (*gaṇitādhyāya*, Chapter 12) of the same treatise.

Qu Anjing 曲安京 (Northwest University, Xi'an, China)

Two cases of reasoning in Chinese mathematical astronomy

In this talk I will discuss a piece of text in which we can find how people in Han dynasty (1st century) to determine the constant of tropical year by a mathematical reasoning. And according to a very short message in Gengwu epoch calendar (1220), we may find how Yelv Chuchai (1190-1244) reasoned the size of Earth.

Tuesday, December 5th, 2017

Role of diagrams and other artefacts in reasoning

Chair: Christine Proust

Reynaud, Adeline

What diagrams tell us about practices of mathematical reasoning in Old-Babylonian Mesopotamia The example of the approximation procedure in tablet YBC 8633

Abstract – On a fairly large number of Old-Babylonian clay tablets containing the statement and the resolution of mathematical problems, one or more diagrams have been drawn in relation to the discursive text. The aim of this contribution is to shed light on the variety of roles that may be assigned to these visual aids, and to illustrate the way in which their observation can help us to understand some of the mathematical practices linked to the production of the documents in which they are contained. The example of an Old-Babylonian (ca. 2004-1595 BCE) tablet of unknown provenience that is now stored in the Yale Babylonian Collection in New Haven and contains an approximate determination of the area of an isosceles triangle, YBC 8633, will enable us to explore various aspects of this issue : by describing minutely the diagram drawn on this tablet and confronting it to the discursive text, we will analyze to what extent it may have been used as a working tool by the author and the readers of the tablet, how it may have served the working out of the problem, and what it reveals on reasoning practices based on impossible configurations.

Husson, Matthieu

Multifaceted mathematical reasoning in Jean des Murs' De moventibus et motis (1343)

Abstract – Reasoning is a central concern in many mathematical traditions and it is expressed in various sources reflecting different practices. When we address mathematical reasoning in mathematical sciences, taken broadly, the diversity of mathematical reasoning becomes greater in relation to the variety of contexts in which those reasonings were shaped. Sometimes actors use the possibility of weaving different levels of reading of the same reasoning according to the rhetorical form chosen to express it. Johannes de Muris' Jean des Murs' *De moventibus et motis* is a clear instance for this. The first part of the fourth book of the *Quadripartitum numerorum*, the text is known for several reasons in the historiography. A chapter on spirals, Archimedean in flavor, attracted Marshall Clagett's attention. Remarks in it about incommensurability and celestial motions were put in relation with similar arguments that Nicole Oresme made later. Finally, historians of natural philosophy analysed Jean des Murs discussion of Bradwardine's conclusion on

the ratio between the velocities of mobiles. Our aim in this paper is to examine the mathematical reasoning expressed in the *De moventibus et motis* and to show how Jean des Murs uses the possibility of the specific form he chooses for his text to extend its core mathematical meaning in the direction of astronomy, cosmology and natural philosophy.

Zhou Xiaohan

Methods using duan – 段 segment [of diagram] – in Yang Hui’s 楊輝 (fl. 13th century) works

Abstract – In the third century, in Liu Hui’s 劉徽 commentaries on *The Nine Chapters of Mathematical Procedures* 九章算術, the commentator has used the rearrangement of diagrams (*tu* 圖), in the form of material objects (Chemla 2010), to carry out reasoning related to the mathematical methods contained in the book. This use of diagrams obeys a rule, to which modern scholars usually refer as “out-in principle” (*churu xiangbu* 出入相補原理). In the 12th and the 13th centuries, another term, that is, *duan* (段 segments [of diagram]), appears in several works presenting reasoning about procedures. The way of using *duan* seems to derive from the use of *tu* in Liu Hui’s commentaries. Even though Liu Yi’s 劉益 work is not extant, his texts or diagrams that involve *duan* are quoted in Yang Hui’s works. In this paper, I will address the use of *duan* in Yang Hui’s *Xiangjie jiuzhang suanfa* (Detailed Explanations of the Nine Chapters on Mathematical Methods 詳解九章算法 1261 C.E.) and *Tianbu bilei chengchu jiefa* (Quick Methods for Multiplication and Division [for the Surfaces of] the Fields and Analogies 田畝比類乘除捷法 1275 C.E.), to examine where and how *duan* is used in the procedures. I will also examine how we can interpret the *gou gu shengbian shisanming tu* (勾股生變十三名圖 Table of the thirteen items which the base and height of a right triangle generate and into which they change). Scholars have noticed the existence of a method related to *duan* and commonly called *yanduan* (演段 deducing the segment). They have carried out research on it at different periods. In addition to reexamining this method, I will also focus on the concept and operation of *bianduan* (變段 changing the segment), to disclose the difference between the two in processes of reasoning with respect to mathematical methods.

Wednesday, December 6th, 2017

Mixing cultures of reasoning

Chair: Matthieu Husson

Chen Jiangping Jeff

Persuasion and Refutation in Fangcheng lun – A Case Study of the Practices of Reasoning in a 17th-century Chinese Algebraic Treatise

Abstract – The issue of mathematical reasoning in late imperial China has been a stimulating topic in the history of Chinese mathematics. The publication of *Jihe yuanben* 幾何原本, the Chinese translation of the first six books of Euclid’s *Elements* in 1607, upended the practice of excluding reasoning in the mathematical works, a peculiar tradition for at least the previous two centuries. In comparison, by the second half of the 17th century, almost all mathematical treatises in China contain explanatory texts as

reasoning in the main text. Many geometric texts followed the style in *Jihe yuanben* of presenting mathematical properties in postulates, definitions, statement of theorems, and proofs. Geometry as a mathematical subject in 17th-century China was considered of “Western” origin. The algebra or computational methods, on the other hand, were believed to be “Chinese” and therefore did not follow the geometric model in terms of presenting arguments and reasoning. This article focuses on the practice of presenting reasoning in an algebraic work *Fangcheng lun* 方程論 (On Measuring through Juxtaposition), composed in 1670s by one of the most prolific mathematicians in Qing China (1644-1911), Mei Wending 梅文鼎 (1633-1722). The subject matter is equivalent to modern-day systems of linear equations and Gaussian eliminations in linear algebra. In traditional treatises composed during Ming China (1368-1644), *Fangcheng* problems were classified according to the number of unknowns involved; and the corrected answers were found by applying two sets of operations, the selection of which might depend on the number of unknowns or the positions of equations being operated on. The scarcity of explanation made it impossible to make sense of the rules governing operations. Instead of following the practice in the traditional texts, Mei re-categorizes the *Fangcheng* problems according to the signs of the coefficients in the initial setting. He only utilizes one set of operations and ignores the other to solve all the problems, supplementing rules and ample explanation to legitimize his “innovations.” Mei’s narratives and texts of explication are meant to persuade his contemporaries that his non-conventional approach can solve “all” solvable *Fangcheng* problems. Moreover Mei refutes certain incorrect and un-explicated practices followed by the Ming scholars and his contemporaries. To further systemize the treatment of *Fangcheng* problems, Mei addresses the fundamentals of setting up positive and negative numbers at each stage of the solution process, discusses the numbers of “computations” needed for each kind of generic problems before achieving the correct answer and various scenarios which result in fewer steps, and clarifies the applicability of *Fangcheng* solution to other genres of problems, as well as its limitation. Well aware of the fact that his innovative tactics deviate from the existing traditional practice, Mei appeals to the antiquity through the notion of *suanli* 算理 (Principles of Mathematics) to promote his treatment and ultimately defy the tradition. His explanatory texts and narratives are to illuminate the hidden *suanli* in his prescribed operation and rules. Once his system of solving *Fangcheng* problems is “shown” to conform to *suanli*, it must be, Mei contends, compatible with or could have been the approach in antiquity. Being included in the compendium resulted from the imperial editorial project to include all things mathematics in the early 18th century, *Essence of Numbers and their Principles, Imperially Composed* (*Yüzhi shuli jingyun* 御製數理精蘊), and consequently becoming court-sanctioned, Mei’s classification and rules of operations were followed by Qing mathematicians of the next generations and effectively elevated to be the orthodox approach to solve *Fangcheng* problems in the 18th and 19th century in China.

Li Liang

The perceptions of Western and Islamic terminology for astronomical tables in early modern China

Abstract – Terminology opens a window on actors’ understanding of specific domains and on the kind of reasoning they build within them. The transformation attested to in the astronomical terminologies that relate to different calendrical systems in China reveals changes in the perceptions of astronomical computations and of their relations to geometry. The traditional Chinese astronomy had its own terminological system for

different calendrical systems. When the Islamic astronomical tables were introduced and adopted in early modern China, some new terms were created. Even though the Chinese users could operate these tables without the obstacle of the unfamiliar terms, they didn't have a clear understanding of these terms and the theories behind them. After Western astronomical tables were transmitted to China, some scholars tried to establish a connection and made an analogy between the traditional Chinese terms and the new Western terms. In addition, they sought to understand the obscure Islamic terms with the help of Western astronomical theories, when they obtained more grounding geometrical knowledge.

Crozet, Pascal

Abū Kāmil's algebraic proofs – La démonstration algébrique chez Abū Kāmil

Résumé – Déjà présente en filigrane dans l'oeuvre d'al-Khwārizmī, la distinction entre démonstration géométrique et démonstration algébrique se fait beaucoup plus nette et explicite chez son successeur Abū Kāmil. L'ensemble des problèmes résolus dans son Algèbre montre en effet une grande variété de méthodes qui montre souvent une volonté nette de s'affranchir de la tutelle de la géométrie.

Ce qu'Abū Kāmil nomme “démonstration par l'indication”, et qu'il oppose explicitement à la démonstration géométrique, n'en est que l'un des exemples. En s'appuyant sur l'étude de quelques problèmes, cette communication voudrait précisément rendre compte de ces premiers développements au sein d'une discipline alors en plein essor.

Thursday, December 7th, 2017

Meaning of steps and accounting for the correctness of procedures

Chair: Agathe Keller

Proust, Christine

Reasoning running through a series of problems. An analysis of some procedure texts from Mesopotamia, early second millennium BCE

Abstract – Mathematical cuneiform problems are generally composed of a statement followed by a procedure, which, in turn, is composed of a succession of steps. The meaning of the whole procedure emerges from the meaning of each step. However, the problems rarely appear isolated. Rather, most often, they belong to sets of problems gathered in one clay tablet. Sometimes, the meaning of the procedure cannot be detected at the scale of one individual problem, but it can be understood by considering the whole set of problems it belongs to. This paper focuses on a catalog of 31 problems dated to the Old Babylonian period (early second millennium BCE), and two procedure texts which make clear the steps of the algorithm solving some of these problems. The following issues will be discussed. What is the scale of the reasoning? That is, is it that of the individual problems, that of groups of problems, or that of the whole series of the 31 problems, or finally is it a combination of these scales? What is the meaning of the steps in the procedures? Is this meaning transparent? Some problems seem very similar, and differ only on tiny details. What is the meaning of these tiny differences? What do they tell us about the reasoning?

Zheng Fanglei

Reasoning with the participation of unknowns before they are found-demonstrative texts in Al-Khwarizmi's Algebra and Nemorarius' De numeris datis

Abstract – Problems of finding numbers are the subject of many mathematical works of Arabic and Latin authors. In these works, the solutions of problems are usually presented as a series of (instructions of) calculations starting from the known numbers, which we call “algorithms”. Nevertheless, besides the algorithm, the authors often write down some texts, apparently reasoning, on the problem they are solving. Such discourses could be regarded as “algebraic transformations”, just like the way they used to be interpreted. However, this way of interpretation has little help for us to understand what role the authors want the reasoning discourse to play in the exposition of a problem and the importance of it in the history of mathematical reasoning.

I shall take examples from two classical works—Al-Khwarizmi's Algebra and Jordanus Nemorarius' De numeris datis—to show how the two mathematicians both take the unknowns into the reasoning, though develop it in different ways. By observations of details, especially in the comparison of the reasoning and the algorithm of a problem, I shall argue that the reasoning discourses in these two works demonstrate rather how the solution (algorithm) is found than why it is correct. Finally, by the comparison of its function to the demonstration part in a problem presented in Greek arithmetical tradition, I shall open the question whether we can regard these reasoning discourses in question as a sort of mathematical proof.

Chemla, Karine & Zhu Yiwen

Algorithms carrying out derivations (tui) versus Algorithms for looking for (qiu)

On the first entry of Mathematical Procedures for the Five Canons

Abstract – In *Mathematical Procedures for the Five Canons*, Zhen Luan 甄鸞 (fl. ca. 570) comments on chosen passages from the Confucian canonical literature. To do this, his annotations take the forms of “methods (fa 法)”. In his own words, some of these “methods” aim at “deriving” (tui 推) a result, whereas others aim at “looking for (qiu 求)” it. How can we interpret this difference? How does this difference relate to commentators' use of it in the context of other mathematical canons? These are some of the issues that we address in this article. In 656, Li Chunfeng 李淳風 (602-670) and his colleagues completed a subcommentary on *Mathematical Procedures for the Five Canons*, in the context of editing and annotating canonical literature in mathematics. How did they comment on Zhen Luan's mathematical procedures depending on whether they were “deriving” or “looking for” a result? This will be the second issue that this article deals with.
